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ABSTRACT

Since its inception, Project Mathematics Anxiety Reduction Training (Project MART) has assisted community college students enrolled in career programs attain sufficient skill and confidence in mathematics so as not to limit their occupational options. Given the realities of the community college, MART has attempted to inform faculty about the role math anxiety plays in conditioning student attitudes and responses and about ways in which faculty might modify classroom interactions to reduce anxiety. Following a brief summary of the first two phases of the project, this document focuses on the third phase. The major goal for this phase was to encourage adoption of in-class solutions to students math anxiety-avoidance behavior through a model workshop targeted at community college faculty and to continue to support, through the development of instructional materials, faculty already motivated to consider new pedagogical approaches. Major areas addressed include: project background, preparation for phase 3, group leader training, workshop implementation strategies, and MART assessment. Following a summary/conclusion section, a 77-page leader's guide, faculty follow-up questionnaire, and bibliography are presented in appendices. The leader's guide, intended for mathematics instructors wishing to conduct staff development workshops focusing on issues on mathematics anxiety/avoidance, contains materials necessary to conduct such a workshop. (Author/MP)

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PROJECT MART (MATHEMATICS ANXIETY REDUCTION
TRAINING) III:

ENCOURAGING THE DISSEMINATION AND USE OF
INSTRUCTIONAL MATERIALS TO COUNTERACT
STUDENT MATH AVOIDANCE

BARBARA R. HELLER

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IN COOPERATION WITH THE:

BUREAU OF GRANTS ADMINISTRATION
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FOREWORD

This document, representing the third and final year of the Project MART undertaking, is presented with great pride. It describes the very successful culmination of a unique approach to a problem about which more and more is finding its way into the literature.

Unlike other approaches which seek remediation of the math anxiety/avoidance syndrome by counseling with students, MART sought to counsel teachers to minimize behaviors on their parts which might contribute to the syndrome and also to, hopefully, positively impact many more students through them than could be reached in a direct student service mode.

As will be seen, the project trained leaders of one CUNY and four SUNY community colleges in the concepts and procedures for overcoming math anxiety and the related avoidance behaviors. Subsequent to our interventions three colleges implemented the program with other faculty members, and in all instances the entire faculties of the math departments engaged in the process after working hours, on a voluntary basis. Of equal or greater significance for proximate evaluation is that three of the colleges formed committees to go beyond the program and conduct follow-through activities and evaluation.

Finally, characteristically, a Leader's Guide is presented for dissemination, which is an additional contribution to the literature and to the process.

Readers are invited to share their thoughts, experiences, and/or reactions with the author, the Institute, or the New York State Education Department's Bureau of Grants Administration.

Lee Cohen, Ph.D.
Director

ACKNOWLEDGMENTS

Unfortunately, writing acknowledgments has become like tipping in a restaurant: almost obligatory. Not so these--the quality of the people, their generosity, interest, productiveness, and outstanding contributions to Project MART make these "thank yous" both genuine and pleasurable.

Many of the same people and organizations that initially played a part in this project continued to have input in its final VEA-funded year: the Bureau of Grants Administration of the New York State Education Department (Mike Van Ryn, Chief and Lawrence E. Gray, Project Liaison); the Center for Advanced Study in Education (Bert Flugman, Acting Director), the Institute for Research and Development in Occupational Education (Lee Cohen, Director and Melanie Bentley, Administrative Assistant)--provided crucial back up.

The colleges that participated in the current year's project deserve special mention. In the body of this report, reference has been made to the support and warm reception MART (and its staff) was accorded at Borough of Manhattan Community College (then Mathematics Department chair, Dr. Lawrence Sher), Onondaga Community College (Richard Hartt, Mathematics Department chair), and Westchester Community College (Louis Rotando, Mathematics Department chair and Sean Fannelli, then Dean of Academic Affairs). The success of the program was due in no small part to the administrations' interest and, I believe, that MART's outcomes reflect and return, in full measure, the hopes they had for it.

The keystone of MART III is its Leaders. All volunteers, Dr. Patricia Wilkinson (BMCC), Professors Ann Tierney, Bernadette Kocyba, and Elana Levy (OCC), and Professors Judith Langer and Henry Korn (WCC) accepted a potentially stressful assignment, for which training was minimal, but which they performed with grace, aplomb, and good will. They were top-rate Leaders and, I suspect, are as warm, caring, and professional in their classrooms.

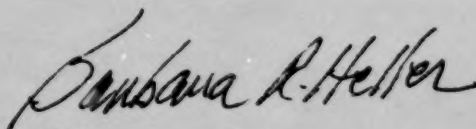
Project staff also had a very active part in furtherance of MART III. Ms. Wendy Geringer, Ms. Cheryl L. Johnson, and Ms. Linda Gross were responsible for organizing the implementation, conducting its evaluation, and drafting the Leader's Guide. They were also instrumental in revising the handbook. MART has benefited from their long-standing involvement, as I have personally.

Ms. Karen O'Brien was the project's Administrative Secretary. In addition to keeping the books, records, and accounts in order, she typed all the manuscripts and forms; the fine quality of her work adds immeasurably to the material's worth. It is no

small feat to type--and retype more than once--those strange symbols and words that constitute the language of mathematics.

Dr. Stan Kogelman, Director of Mind Over Math, was and remains MART's technical advisor. He has done more for Project MART than is generally asked of consultants, and the program bears his unmistakable style. Over the years, he has become more "staff" than hired expert and his interest in MART is familial. Stan's interest in math anxiety extends to the severity of the challenge presented by my own, and he has been both friend and teacher to me.

Dozens of other people--too many to mention--have had a hand in designing and disseminating MART: participating faculty and others who have helped develop the materials. Our combined efforts can be seen in many of the very best classrooms in New York State. I am very proud.

A handwritten signature in cursive script that reads "Barbara R. Heller".

Barbara R. Heller,
Project Director

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**PROJECT MART (Mathematics Anxiety Reduction Training), PHASE III:
Encouraging the Dissemination and Use of Instructional Materials
to Counteract Student Math Avoidance**

OVERVIEW

Starting in 1979,¹ the CASE Institute for Research and Development in Occupational Education (CASE/IRDOE) began to explore ways to deal with the math anxiety-avoidance problems of presumably large numbers of community college students. We wanted an approach to the problem that was efficient and effective, that placed few demands on either the student or the educational institution, that was more skills-based than therapeutic, and that--if successful--could be readily disseminated. The result of three year's work is an in-service workshop program with materials for two-year college mathematics faculty and faculty of math-related subjects that addresses students' negative attitudes toward math by focusing on in-class teaching strategies and practices.

The first-year project, conducted during 1979-80, was directed to faculty from primarily the CUNY (City University of New York) colleges.² We investigated different content, format, program length, and other variables to design a package that would meet our criteria and have applicability to the needs of a wider constituency of participants. Two workshop series were run, producing materials and organizational recommendations that were the basis of the current third-year (1981-82) program.

During 1980-81, with a second grant from the State Education Department, project staff, assisted by experts in mathematics and math education, developed a full set of instructional resources to be used by mathematics professors either individually or as part of an in-service program.³ The materials included readings, essentially in book form, and some supplementary activities, exercises, and examples.

¹VEA #80-3E-56GPS.

²Heller, B.R. et al. Project MART: Mathematics Anxiety Reduction Training. CASE Institute for Research and Development in Occupational Education, Graduate School, City University of New York. CASE #22-80, November 1980.

³VEA #81-53-0392. Heller, B.R. Project MART: Phase II. CASE Institute for Research and Development in Occupational Education, Graduate School, City University of New York. CASE #06-82, May 1982.

The third and final VEA grant allowed us to conduct model staff development workshops in two-year institutions in New York State,¹ using the instructional materials and a Leader's Guide which was designed to aid colleges in replicating the program. This report describes the 1981-82 workshops conducted at three community colleges and presents the followup plans of these institutions and the implementation plans of two others. The major appendix, Appendix A, is the Leader's Guide. Not included is the MART handbook which was revised for the second time during 1981-82 and which will be available as a separate document.

BACKGROUND

Instructors of math and math-based courses required for career programs at the community colleges are beset by more than their share of problems in the classroom. Very many of their students do not like mathematics; some are actively hostile to the subject. Large numbers come without the requisite preparation: in any given classroom perhaps no more than a handful of students has truly mastered the basic arithmetic computations and concepts they need to know to be able to cope with the content of the course in which they are enrolled. Inattention, "tuning out," skills deficiencies are commonplace occurrences portending the presence of math anxiety. If indeed math anxiety is operating in these instances, the behaviors themselves effectively preclude the possibility of improved communication between student and instructor. To facilitate learning it is first necessary to reduce anxiety.

Although the genesis of math anxiety and its handicapping manifestations can legitimately be traced to earlier times and other places, "if only" thinking will not help community college math faculty solve the very real problems they have inherited. To teach students as they find them (ill-prepared, hostile, turned-off), instructors face the solitary task of devising ingenious ways to establish the climate and dialogue that will enable learning to take place. Many college professors, despite their own very high level of achievement in mathematics, are ill-equipped for this task.

Rationale

Research findings, which indicate the widespread existence of math anxiety among students and the general public, together with decades of trial-and-error

¹ VEA Contract #53-82-0287.

practice, have identified a number of factors that contribute to the negative attitudes and misconceptions about mathematics that make it such an anxiety-producing and difficult matter for so many people. In turn, knowledge of these factors (and of the misconceptions themselves) provides a basis for fashioning new approaches to teaching math in ways that diminish student anxiety.

To date, the typical response to combatting math anxiety among college-aged students has been the math clinic. Established on many campuses throughout the country, these clinics use teams of guidance counselors and math instructors to work with small groups of student volunteers. Clinic programs started at prestigious private colleges and universities and, until recently, the bulk of the evidence concerning math anxiety came from the experiences of these institutions.

There is little question that math anxiety is also prevalent among students enrolled in public two-year colleges. There is some data that suggests that it may be particularly widespread among this population, who also bring to the classroom situation other problems and attitudes that contribute to the severity of the math anxiety phenomenon.

In February-March 1980, during Phase I of Project MART, we tested some 600 students enrolled in 35 different math courses at four CUNY community colleges and one private four-year college in New York City. Included were remedial students and students taking regular math courses ranging from basic-required to advanced-elective. Student responses to the Mathematics Anxiety Rating Scale, Form A¹ confirmed the existence of moderate- to high-levels of math anxiety among students in all classes, and particularly among those in need of remediation.²

With math anxiety as extensive as these data indicate, the therapeutic math clinic model is not an efficient one for community colleges to adopt: as effective as this interpersonal approach is, few community colleges can afford the resources necessary to staff and maintain the scope of program that would be required to serve all students in need of help.

Project MART is an alternative. Its focus is on student-teacher interactions and its locus is the college classroom. If, as the literature suggests and our experiences confirm, many of the factors associated with the formation of negative math attitudes and damaging misconceptions grow out of classroom experiences, the classroom

¹Suinn, R.M. Mathematics Anxiety Rating Scale, © 1979. Published by the Rocky Mountain Behavioral Science Institute, Inc.

²Heller et al., 1980. op. cit.

represents an appropriate site of redress, and the instructor becomes the key to bringing about change on a large scale.

Development of Project MART

Since its inception, Project MART's purpose has been to assist community college students enrolled in career programs attain sufficient skill and confidence in mathematics so as not to limit their occupational options. Given the realities of the community college, especially large, urban ones, MART has attempted to inform faculty about the role math anxiety plays in conditioning student attitudes and responses and about ways in which they--faculty--might modify what takes place in the classroom in order to reduce anxiety and thus ultimately improve the chances for learning.

Phase I. Phase I of Project MART, initiated in academic year 1979-80, was a pilot in-service program that focused on (a) increasing instructors' awareness of and sensitivity to students' feeling about math, (b) providing a structured opportunity for faculty to explore their own attitudes and classroom behavior as these might affect student reactions and performance, and (c) demonstrating new pedagogy that could be adapted to most classrooms. Two cycles of staff development workshops were conducted, varying in organization and format, and involving a total of 30 faculty members (28 math instructors, one physics instructor, and one guidance counselor) from five colleges (four CUNY community colleges and one private four-year college) in New York City.

It was not the objective of the first phase to measure what changes in instructional behavior, if any, the experience prompted on the part of the participants or how these changes might have begun to affect their students. What Phase I intended and did establish was that:

- community college students (regardless of level of performance) have moderate- to high-level anxiety;
- math anxiety correlates with poor preparation in math and interferes with achievement;
- without an adequate background in mathematics, students'--especially women students'--career options will continue to be restricted;
- community college faculty and staff need to better understand how math anxiety operates so as to be able to teach more effectively; and
- faculty want, need, and welcome more information about the determinants, manifestations, and implications of math anxiety and more practical

suggestions that they can apply to their classrooms: innovative and anxiety-reducing ways to teach specific mathematical concepts and calculations, improve students' accuracy and retention, manage a classroom, and help students appreciate the importance of mathematical competency in today's world.

Reporting on Phase I, we made the following recommendations:

1. that written materials be developed--materials around which community college mathematics departments could initiate MART-type staff development programs or which individual instructors could use for self-instruction;
2. that additional restructuring be done on the MART in-service workshop model so as to better serve the needs of more varied departments and faculty.¹

Phase II. The second phase of Project MART was implemented during 1980-81 and enabled us to carry out the first of these recommendations: to produce written materials in concert with teams of consultants. The first version of the MART handbook was completed earlier this year², and contained sections on math anxiety and several sections devoted to exploring faculty expectations. Most of the handbook was comprised of classroom management strategies (e.g., testing, the first class session) and specific techniques for teaching fractions, signed numbers, algebra, word problems, and calculus. Many of the materials (exercises and examples) tried out in the initial workshop program were also revised and incorporated.

The resulting handbook, which had limited dissemination, over-emphasized the CUNY experience (which proved to be atypical across New York State). Furthermore, the authors questioned its utility as both a self-instructional guide and as the cornerstone of a staff development program.

Nonetheless, these materials represent the only collection of information about math anxiety among college students that has direct applicability to the instructional process. Select readers and reviewers rated them to be useful, provoking, and enjoyable, despite their limitations. A second revision of the handbook based on a full-scale try-out of the workshop program was one of the objectives of Phase III.

Phase III. The third and concluding phase of Project MART took place during 1981-82. The major intent was to restructure and strengthen the MART in-service program on the basis of the first-year pilot, incorporating the insights we gained

¹Heller et al, 1980, op. cit.

²Heller, 1982, op. cit.

during the compilation of the MART handbook. Important, but somewhat subsidiary, was a concern with making the handbook and the staff development model compatible, and with producing a Leader's Guide that would enable institutions to conduct workshop series with a minimum of external assistance and support.

A good staff development program, however, remains crucial. Materials alone, no matter how good, are virtually useless unless they are brought to the attention of the people for whom they were intended. The MART materials are no exception to this generalization. Indeed, many of our early impressions concerning the attitudes of postsecondary level faculty toward math anxiety persist--and have been reinforced: without collegial pressure, faculty tend to resist the responsibility the MART skills-based approach entails, and are initially reluctant to try the new techniques. Staff want and need the opportunity to interact with the materials and with respected colleagues in an active give-and-take manner.

Little has happened between 1979, when we first began the project, and now to alter the basic premise of the MART program. Students continue to exhibit inadequate skill in mathematics which, in turn, continues to limit their occupational paths, especially those of women in the sciences and technologies. More and more evidence has been accumulating that relates poor performance in math to attitudinal and experiential causes, and the concept of math anxiety-avoidance is being more widely accepted. While math clinics are proliferating on college campuses, however, except for MART, there has been little change or increase in programs based in the classroom.

Project MART, PHASE III, PREPARATION

In this section of the report, we will describe the goals, objectives, and the activities we undertook preparatory to the conduct of the workshop series.

Objectives

Our primary goal for MART: Phase III was to encourage the adoption of in-class solutions to students' math anxiety-avoidance behavior through a model workshop targeted at community college faculty; and to continue to support, through the development of instructional materials, the interest of faculty who are already motivated to consider new pedagogical approaches but who lack specific information

about how to moderate student anxiety and establish an environment more conducive to teaching and learning.

The specific objectives for 1981-82 were to:

1. Restructure the MART in-service workshop model to
 - make it more applicable to a broader constituency, and
 - focus it more specifically on the why to and how to modify teaching strategies;
2. Encourage the trial of the workshop model and its institutionalization by
 - aiding two-year colleges in New York State set up a staff development program, and
 - providing the participating colleges with materials and other forms of technical assistance;
3. Produce a Leader's Guide and a revised participant handbook that would provide other colleges with a basis for conducting staff development workshops in math anxiety-avoidance reduction, through
 - monitoring and assessing the 1981-82 workshop series,
 - evaluating the reactions of participating faculty, and
 - revising the written materials and workshop model to better accord with actual field conditions.

Project Planning

The initial planning stage of this project was spent in structuring a workshop design--specifically, in detailing the content to be covered and the order of presentation; identifying potential sites; and developing the Leader's Guide.

Workshop schedule and content. Preliminary discussions focused first on an optimal length of a staff development program and on different formats. Ideally, there was consensus that a thorough program should consist of 16 to 20 hours of time--eight to ten two-hour sessions once a week. This design was compromised as a result of the opinions of faculty, MART II consultants involved in this year's plans, who believed that this design represented an extraordinary and unrealistic commitment on the part of instructors. The length was set at 12 hours (six two-hour sessions spread over a six-week period), with an optional two hours of additional meeting time that participants could spend on select topics (e.g., other types of programs operating at other sites) or on formulating followup programs for their own institution.

As we will see below, only one of the three colleges implemented this design as given: the other two sites conducted the program more flexibly, although one used the total of 14 hours. The variations in implementation largely reflected end-of-year scheduling constraints: in each and every instance, participating faculty proved more than willing to attend these many workshop sessions and, moreover, to spend the extra time required reviewing the materials. Furthermore, in all cases, the participants agreed to a followup program (to be conducted during 1982-83) that would require additional effort on their parts.

Once the program's length and basic structure were outlined, we turned our attention to the content. This involved reviewing reactions to the MART I program and MART II materials, and identifying redundancies as well as gaps in information. There was a large amount of material to go through; staff and consultants selected the following topics for inclusion, based on our assessment of faculty need and interest. Below is the final syllabus of topics, in the recommended sequential order, complete with an estimate of the amount of workshop time available for discussion.

WORKSHOP SESSION #1

- Welcome (15 minutes)
 - Registration
 - Faculty Opinion Survey
- What is math anxiety? (15 minutes)
 - A brief history
 - A working definition
- Who's got math anxiety? (30 minutes)
 - A brief review of the research literature
 - How to recognize math anxiety:
 - formal measures
 - informal measures
- My math anxiety (40 minutes)
 - The Toy Boat problem
 - Reactions and solutions
- Word Problems (20 minutes)
 - Introducing the Direct Translation Method

WORKSHOP SESSION #2

- Welcome (15 minutes)

- More about word problems (45 minutes)
 - Translating the Direct Translation Method
 - Pros and cons
- The public's image of mathematicians (20 minutes)
 - How others view us
 - How we view ourselves
- Mathematics: what is it? (40 minutes)
 - Students' beliefs and perceptions
 - Mathematicians' beliefs and perceptions

WORKSHOP SESSION #3

- Welcome (10 minutes)
- Characteristics of math-anxious students (30 minutes)
 - Visible symptoms
 - Etiology and effects on learning
- Fractions (45 minutes)
 - A significant problem
 - Different teaching strategies
- Do's and Don'ts in the classroom (30 minutes)
 - General tactics
 - Specific techniques
 - Topics for classroom discussion

WORKSHOP SESSION #4

- Welcome (10 minutes)
- Math myths and their consequences (30 minutes)
 - Myths people believe in
 - Implications of misconceptions about how math is done
- Signed numbers (45 minutes)
 - A significant problem
 - Different teaching strategies
- Topics for classroom discussion (30 minutes)
 - Math is a cumulative subject
 - Math is an acquirable skill
 - Math study habits
- Closure (5 minutes)

WORKSHOP SESSION #5

- Welcome (10 minutes)
- General teaching practices (60 minutes)
 - Expectations and actuality
 - The classroom environment
 - The first day of class
- Calculus (30 minutes)
 - The concept of limits
 - Leaps of faith
- Closure (5 minutes)

WORKSHOP SESSION #6

- Welcome (5 minutes)
- Some hints for teaching algebra (20 minutes)
 - What we do and why
- Testing/grading practices (45 minutes)
- Selecting a textbook and getting students to read it (15 minutes)
 - Evaluating texts
 - What students need to know to use a text
- Followup activities: where do we go from here? (30 minutes)
 - More meetings?
 - Math clinics?
 - Research?
- End of program "housekeeping" (30 minutes)
 - Faculty Opinion Followup Survey
 - Materials' Rating Forms
 - Thank yous

We tried to vary the content and sequencing to achieve a mix between theory and application. We also attempted to systematically interweave discussion with more active exercises, and to balance remedial-type subject matter with regular mathematics course demands. Overall, our intentions were achieved, but the content retains an emphasis on what appears to be--at first glance--basic arithmetic concepts and manipulations.

Our concern with fractions, signed numbers, and word problems reflects our

experiences that these are the most troublesome topics for the typical community college student, irrespective of whether he/she is found in a remedial or regular math course. Very many people, not all of whom can be labelled or who will label themselves as math anxious, are not truly comfortable with these concepts; math anxious people rarely are and will more often than not attribute the onset of their problems with math to the introduction of fractions, or signed numbers, or to the first "if two trains..." problem.

Because mathematics is so cumulative a subject, failure to master the basic topics and computational rules continue to plague the learner forever after. Students in calculus courses have difficulty with trigonometric functions which can frequently be traced back to difficulty with some idea concerning fractions. Students studying algebra are often at a loss because of incompetencies in dividing numbers with unlike signs.

In the early workshop sessions, participating faculty would question why so much of MART's content was devoted to "elementary school" mathematics. As this issue was discussed, invariably colleagues would describe stories of students having difficulties in required courses--difficulties that had roots in "baby" math. While the content of the MART program may have greater relevance for teachers of remedial students (who generally have higher levels of anxiety), it also has relevancy for teachers of regular math subjects. (And as elementary and secondary math education continues to suffer because of the shortage--growing shortage--of trained and qualified teachers, students can be expected to come to college with increasing problems relating to fractions, signed numbers, word problems--not to mention algebra and calculus.)

While we have retained these math concepts because of our belief in their wide applicability to students, we have chosen to keep this emphasis for other reasons: the approach we propose to these topics, as well as the points at which students have problems grasping them, have generalizability to more advanced subject matter. Thus, for example, the "leap of faith" required for the concept of infinity in calculus has its counterpart "leap of faith" in being able to subtract signed numbers according to the algorithm. It is to the instructor's advantage to be aware of the experiential similarities.

Leader's Guide. Once the scope of the content was defined and a syllabus generated, the development of a Leader's Guide followed. The Guide, which is appended (see Appendix A), was designed to assist mathematics instructors in conducting a MART staff development program.

The Guide is intended to provide an instructor-group leader (a peer leader) with an overview of the workshop program that differs in significant ways from the perspective of participants. First, for each component, detailed Leader objectives are included. Second, the Guide details the time allotment for topics, with the suggested allocations reflecting the desired emphases. It also includes exercises the Leader might introduce in a workshop to supplement those included in the handbook. The primary distinguishing feature of the Guide is its focus on the points the Leader makes and continuously reinforces (such as the anxiety math instructors themselves experience when faced with a challenging mathematical problem). These are spelled-out repeatedly and in detail.

The Guide parallels the content of the handbook, but not the handbook's sequence or tone. The Guide and handbook have somewhat differing purposes: since workshops must be, by definition, more active than "reading a book," the ordering of topics in the Guide is varied to counteract boredom and to elicit participant input; furthermore, to maintain a high level of motivation--and, therefore, good attendance--workshop programs should not provide complete closure at each session or after each topic. Another crucial difference between the Guide and the handbook pertains to the issue of authority: in the MART workshops, the group Leader is a peer leader, not necessarily an expert in the area of math anxiety, nor responsible for the substantive content of the program. The leadership model must preserve the peer relationship and maintain the integrity of the contributions of all participants. To this end, the Guide as well as the Leader Training Program, has the Leader take a more tentative, less dogmatic position than the handbook--and relies instead on the shared experiences of the group for consensus.

Books, on the other hand, and especially books of the "how-to" genre, must be cast more authoritatively to foster believability and acceptance on the part of the reader. While books, too, must maintain the reader's interest, they suffer from fragmentation when the content does not follow a logically developed sequence. Books generally also have to be more directive in terms of tasks set for the reader and must provide the support that a workshop group provides. How-to books rely heavily on the weight of expert opinion.

The Guide was the basic tool of the Leader Training Program (see below) and the Leaders' major aid in conducting their workshop programs. Two of the Leaders and their group referred to it as the "bible," offering it as the "voice of authority." In observing the workshop sessions at each site, the Guide was always in evidence; it was not uncommon for Leaders to read sections of it to participants, especially at times

when the session became unfocused or the relevant conclusions were not being elicited from the participants in more spontaneous fashion.

The Leader's Guide was formatted for easy use. In loose-leaf form and printed on colored paper, it was easily distinguishable from the handbook (printed on white paper) and from the exercises distributed to participants (printed on paper of a third color) during and at the end of sessions. It left a lot of "white" space for Leaders' notes and reminders, and it was cross-referenced to the corresponding chapters of the handbook. The Guide provided the Leaders with a framework for the sessions, with information in addition to that of the participants, and with a handy reference.

Identification of sites. The proposal called for the implementation of MART workshops at four community colleges: three SUNY and one CUNY institution. Responsibility for identifying institutions that would be interested in and willing to undertake the program was left to the project director, with substantial input from the Bureau of Grants Administration.

We wanted schools to represent different regions of the State and which served diverse student populations. Some variability in the size of the math department--in the number of full- and part-time faculty and adjuncts--was also desirable. Willingness to participate was an important factor, since the colleges were being asked to make commitments of resources and money. Indeed, not only did the institution have to make space available for meetings and to help arrange a time during which the math faculty could meet, but they were also asked to provide refreshments as well as help offset the expenses incurred as part of the Leader's training.

Starting with a list of four colleges that met these criteria, our liaison at the State Education Department wrote to these colleges' Presidents describing the mathematics anxiety reduction program. These December 1981 letters were followed up first in writing (in January 1982), and then by phone when project staff contacted the respective chairs of the math departments, offering more details about the intended project.

In all cases, the chairs had been informed of our interest and introductory meetings were set up. The project director and consultant met with representatives of the math department and with the math chair of Borough of Manhattan Community College on February 10; with math faculty and administration of Onondaga Community College on February 17; with the math chair and faculty of Schenectady County Community College on February 19; and with Westchester Community College administrators and math faculty on March 1, 1982.

Site Selection and Orientation

We approached each college with the expectation that it would be participating in the program. Our introduction had been well-received and, with the acceptance of mutual responsibilities, only the selection of a Leader, dates for training, and a preliminary workshop schedule needed to be worked out.

Borough of Manhattan. In addition to project staff (the director and consultant), present at the first meeting at Borough of Manhattan Community College were the department chairman, chairwoman elect (and a MART II consultant), and two senior members of the mathematics department. The philosophy of MART was addressed and attention was given to the mechanics of the program operation.

Despite some initial hesitancy on the part of the faculty members who thought their colleagues would be reluctant to volunteer the time required by this staff development program, a Leader was identified (the elected chairperson) and tentative times for the workshops was agreed on: because the college was preparing to relocate to a new physical plant and because the faculty was teaching in several different buildings, the chairman offered MART the time (and place) usually devoted to departmental meetings. It was agreed that the Leader would use a departmental meeting for the first workshop session after which, and depending on the number of interested persons and their schedules, a better arrangement might be made.

The meeting was concluded with the selection of a Leader and her commitment to attend a two-day training program in March, an agreement to implement the MART program in the late Spring, and a time and place that was set aside for at least the first session. Enthusiasm for the program was mixed, with the administrators more optimistic than the faculty representatives about its appeal to the mathematics faculty.

Onondaga. The project director and consultant met with the entire mathematics faculty, including the chair and personnel from the math lab in mid-February. Despite the fact that a Leader was chosen (actually, three co-leaders) and was available to come to New York City for the training program, the orientation did not go smoothly. Project staff had not prepared materials for distribution and their description of the program's objectives and tactics did not meet the faculty's need for specificity.

Nonetheless, there were no particular problems raised and it was agreed that Onondaga Community College would conduct the MART workshop program. The department chairman offered the use of a faculty lounge and refreshments for the

sessions, and his support to the three faculty members who had volunteered for the Leader Training Program.

Westchester. At the beginning of March, project staff was invited to Westchester Community College to meet with the Dean of Academic Affairs, the mathematics department chairman, two faculty members who were experienced in math anxiety (one who had been a MART II consultant and the other who had been involved in the math component of a career development program run by the author), and four other faculty representatives.

The meeting ran smoothly, spurred in large measure by the Dean's interest in the program. Faculty objections, which included a presumed disinterest on the part of a large number of other staff, lack of a suitable space and time for meetings, and additional work for a Leader, were countered with concrete offers of assistance: small stipends to the Leader(s); meeting space and refreshments, including lunch if a one-day, double session was scheduled; and encouragement of faculty participation.

It was decided that the College would use two co-Leaders for the program, both of whom would be trained. Specific recommendations for involving faculty were generated and the orientation meeting concluded on a highly positive note in anticipation of an interesting and exciting workshop series.

Schenectady. The director and consultant visited Schenectady County Community College in mid-February. Present at this introductory session were the chairman of the Mathematics, Science, and Technology Department and the three persons comprising the math faculty. The meeting was arranged by the College's Coordinator for Curriculum Review who shared the President's interest in the program.

One of the three faculty members had volunteered to act as a MART Leader and the day's discussion centered around two issues: whether a sufficiently large group could be recruited from faculty of math-related subjects (a workshop group of from 8 to 12 people is necessary for the kind of interaction MART requires); and how the MART approach differed from and improved upon the experimental curriculum two of the faculty had been developing. Other questions (of space and availability for training) were not seen as potential stumbling blocks and the project team came away from the meeting expecting no implementation problems.

With the apparent commitment of four sites, we proceeded with plans for a two-day training session on March 19 (a Friday) and March 20 (Saturday), 1982. Two days

before these dates, Schenectady College informed us that they would not be participating in this year's program. As a result, at the eleventh hour, we contacted two other community colleges that had expressed interest in the program: Orange County and Suffolk County.

The math department chairs of both Colleges decided that MART had appeal and relevancy to their institutions and identified a faculty member with interest in math anxiety. One person from each school was able to attend the two-day Training Program, and each expressed interest in trying the MART workshops on their campus.

GROUP LEADER TRAINING

As scheduled, the two-day Leader Training Program took place on March 19 and 20, 1982 in New York City. In addition to the project staff (director, assistants, and consultant), also present were the Leaders from Borough of Manhattan Community College (one person), Onondaga Community College (3), Orange County Community College (1), Suffolk County Community College (1), and Westchester Community College (2). Although the initial plan called for one leader from each college, two colleges wanted to operate the workshop program with more than one. Whether or not it was coincidental is indeterminate, but in both these cases, the MART workshop program was better implemented. However, irrespective of this outcome, co-leaders are recommended for the future. MART leadership is complex: the material is sensitive; math faculty is generally unfamiliar with small-group dynamics (as opposed to the dynamics of classroom teaching); and in the MART model, the leader functions as a peer leader, charged with conveying an approach and content developed elsewhere.

Leader Training Philosophy

Preparing faculty for their role as Leaders meant familiarizing them with the theory underlying the MART approach to math anxiety (i.e., its focus on classroom instruction); providing them with a rigorous understanding of the etiology of math anxiety and of its various and often maladaptive manifestations; offering them help in working with colleagues in small groups; and showing them how to use the materials and accomplish the evaluation and other housekeeping details required by this demonstration.

Since we had only two days to achieve these goals--in retrospect, three days

would have been preferable--we decided to provide leaders with a model. In other words, the training was set up to be a replica of the 6-session workshop program during which we would also make explicit certain of our group-leading strategies and how we handled the materials, the problem examples, and the supplementary activities. Therefore, the two days of training were scheduled as follows:

- Day 1 - Project MART background
 - Responsibilities of a MART faculty Leader
 - Workshop Sessions 1, 2, and 3
 - Small-group techniques...

- Day 2 - MART evaluation and other administrative chores
 - Small-group techniques, continued...
 - Workshop Sessions 4, 5, and 6
 - Technical assistance
 - Individual college followup

Each day began promptly at 8:45 a.m. with coffee and refreshments and continued until after 6:00 p.m. Lunch and an afternoon snack were also provided so that participants could work uninterrupted.

The Leader's Guide constituted the basic training tool. As already briefly described, it was intended to be used in conjunction with the MART handbook (and is cross-referenced to the 1980-81 and 1981-82 editions). However, it is also self-contained, consisting of session-by-session objectives, supplementary materials, ordered presentations, and sufficient background information about math anxiety and other substantive topics. The loose-leaf format allowed Leaders' to reorder the materials and the wide margins are useful for notes.

Our consultant, a mathematician renowned for his expertise in math anxiety and familiarity with the MART program and materials, conducted the training. He was assisted by the project director who served as co-trainer. Her role with respect to the math content was largely supportive and her reactions to the discussants provided Leaders with some helpful insights into the thinking and behavior of math anxious learners. Prior to the session, the director and consultant had worked out a set of procedures, building on their past experiences as MART co-trainers. Their approach worked well: it kept the interactions lively and on target and, importantly, stimulated thinking about instructional processes.

Typically, the training proceeded as follows. One or the other of the co-trainers introduced the session, describing its goals and how they related to MART's general

theme. Specific objectives were enumerated and the rationale was discussed. The materials required for the conduct of the workshop sessions were listed as was the kind of preparation recommended for Leaders. Then, the group was put through the session--at a greatly accelerated pace--with the colleges' Leaders taking the part of workshop faculty participants.

The co-trainers would interrupt the flow of activity at irregular times to highlight some aspect of group dynamics. As an illustration, as the group worked on the "Toy Boat" problem, one of the trainers jotted down their random comments. When it came time to discuss their feelings about this exercise, we described why we took notes and how they might do it in the session they would conduct. Also, it was not unusual for a co-trainer to break into the discussion with remarks like, "Notice that when the Leader doesn't know the answer, or doesn't want to answer a direct question, he doesn't respond and this creates a silence that the questioner (or some other member of the group) will fill." Or, "When the Leader wants to elicit others' opinions, notice that she turns slightly in her chair and makes eye contact with an individual."

With this kind of guidance we had hoped to help participants become more responsive to the dissimilarities between leading a small-group and teaching a class. It was especially important that the Leaders begin to appreciate not only the effectiveness of small-group techniques, but the soundness of specific ones that would work exceptionally well with peer groups. We also anticipated that they could adapt this non-directive form of leadership to some classroom situations--to reduce the authoritativeness with which mathematics instructors are perceived. (Any strategy that serves to humanize the instructor will carry over to mathematics itself and should improve the classroom climate necessary for learning.)

We also attempted to anticipate the topics that were likely to generate the most controversy and, to the extent that time permitted us to do so, explore all sides of the arguments. It was here specifically that we stressed that the particular emphases were ours--not theirs (i.e., the Leaders)--and that their task lay in helping participants see many points of view and the potential effectiveness of many different approaches. At those times when there would be disagreement with a suggested technique, we wanted the Leaders to be able to communicate that "that seems like a fine way of teaching that; let's examine first, whether it is an anxiety-reducing or anxiety-producing and second, whether it precipitates problems when we introduce the next algorithm."

Finally, we kept the training informal, hoping that the Leaders in turn would

approach their task with a sense of easy enjoyment. Although we had been able to involve groups of faculty in this fashion in prior years, and believed a light touch contributes to MART's positive outcomes, we also stressed that some of our specific "tricks" might not work as well for them: both in a small group or in a classroom situation a leader-teacher cannot use others' ideas or approaches--no matter what their merit--without adapting them to comfortably fit their own styles.

Assessment of the Training Component

Overall, the training was well rated both by participants and project staff. There were, however, some problems recognized immediately which were subsequently played out on the colleges' campuses.

First, two days of training is insufficient. While the highly compressed schedule heightened intimacy and involvement, and resulted in lasting friendships among and between the concerned parties, it was extremely fatiguing. Moreover, there was not enough time for Leaders to practice or demonstrate techniques. They particularly needed more support and more focus on the dynamics of small-group interaction. The leadership skills required by the MART model differ dramatically from teaching skills. Especially when the group is composed of colleagues, leadership becomes complex, highly sensitive, and difficult. For the most part, the Leaders were effective, but as we observed in the workshops, their inexperience frequently showed: no more so than when the workshop groups discussed any one individual's teaching approaches. Future Leader Training Programs need to attend to conflict resolution skills and to how to "depersonalize" discussions.

More time devoted to the handouts, supplementary exercises, and activities would also have been beneficial. The MART package consists of several pieces; workshop Leaders need more practice in how to use them, how to select appropriate activities (and when to reject alternatives), and how and when to substitute materials of their own devising. We witnessed several instances where MART materials was not used and Leaders' own examples incorporated instead. While on the one hand this is evidence of positive motivation and an attempt to give the program an individual imprint, on the other hand, the substitutions did not always have the desired impact. Had more stress been given to the bases for the selection of activities, as well as to their ordering, Leaders would have had a better understanding of making these decisions.

An additional amount of time would have also assured participants an opportunity to explore college-specific issues, although ideally such discussions should

take place after they had had the experience of conducting a couple of workshop sessions. So too would such a followup session have been useful for working out any problems of co-leadership--despite the fact that in all instances the co-Leaders had planned for the division of responsibilities.

Other than an extension of the training time to two and one-half days or two initial days followed by a half day after the conduct of one or two workshop sessions, neither staff nor Leaders suggested other modifications.

Of note is that, based on this year's experience, some Leadership Training is necessary. We intended to develop a Guide that incorporated all the information and background support necessary to conduct MART workshops; however, it seems quite clear that faculty--even faculty as interested in math anxiety and as sympathetic to the MART focus on the instructional process--even faculty who willingly volunteer for the role as leader and who have the support of their administration--even faculty who are good and experienced teachers--need a training program. In some real sense, MART challenges participants' fundamental beliefs about themselves and about how they function as teachers; it also challenges their experiences as learners and their expectations about their students. To be able to guide participants through a reexamination of these beliefs without doing disservice to any individual member of a group that must continue to work together long after the end of the staff development program, requires preparation of a kind that is only available through a structured, interactive training program.

WORKSHOP IMPLEMENTATION

In the interval subsequent to the Training Program and prior to the end of September 1982 (the end date of the grant), three colleges implemented the MART workshop program: Borough of Manhattan, Onondaga, and Westchester Community Colleges. Orange County and Suffolk County Community Colleges have not (to date) done so, despite assurances that workshop series for the Fall 1982 term were seriously contemplated.

Below, we will describe each site's experiences. General trends will be highlighted, and faculty's ratings of the materials and series will be summarized.

The Implementation Design

As already described, the optimal, "compromised" implementation model consisted of six, two-hour sessions spread over the course of six weeks. A seventh session, for putting into effect followup plans, was strongly suggested. Only one community college, Onondaga, followed this model; the others instituted variations of it.

The recommended size of the workshop group was between 12 and 15, exclusive of the Leader(s). This number, allowing for absences, guarantees a sufficiently large discussion group. Fewer than eight participants are too few to generate the necessary high level of interaction; more than 15 would have similarly limited maximum input by group members. While primary consideration was to go to faculty from the mathematics department, we suggested that Leaders go outside the regular, full-time faculty to fill this quota: first to part-time, adjunct, and/or auxiliary staff (math lab tutors, technicians, etc.), and then outside the department to faculty of the math-based or related disciplines. Despite Leaders' initial hesitancy about recruiting these many interested people, it proved to be no problem.

Interestingly, attendance was also no problem. Eight out of every ten participants at each site attended all of the workshop sessions; in most other instances, an individual missed only one session, usually because of a conflict with the date or time. There were, however, a couple of people in each college who attended the first session or two and who never came back. Invariably these people were described by the Leaders as the "ones who needed MART the most"--the department's poorest instructors or the most anxiety-producing ones.

This is not to say that attendees were always delighted to be there. One participant, we were told, was "afraid not to come"--afraid that his colleagues would discuss him if he wasn't there. However, others who voiced complaints about attending or who (at the start of a session) indicated they would have to leave exactly on time, came and stayed--frequently these individuals were the last ones to leave the meeting, and it often happened that they were the ones who were most reluctant to suspend a discussion until the next session.

Participation in the MART program was voluntary. No stipends or inducements were offered to participants; no penalty, not even subtle ones, was attached to non-participation. Although we had the active and hearty support of the colleges' administration, we discouraged their involvement once the program got under way. Only at Borough of Manhattan Community College did any math department chairpeople attend a workshop--there, the outgoing chair in his role as faculty and the incoming one in her role as Leader.

However, the administrations' interest in the program was outstanding. They arranged for the best possible space for the meetings and for coffee, tea, and snacks. At one full-day session at Westchester Community College, the administration provided lunch as well as morning coffee and donuts and, at Onondaga, dinner was provided when project staff visited. Furthermore, Leaders were urged to keep the administration abreast of followup plans, which the colleges' indicated they would facilitate.

The project director and consultant made observation visits to each of the programs: we visited Onondaga three times, Westchester three times, and Borough of Manhattan twice; we saw approximately 50 percent of the total number of sessions conducted. The primary purpose of these visits was to obtain first-hand information about (a) how well the content of MART met faculty needs, (b) whether the Leader Training Program provided sufficient preparation, and (c) how the materials were used. We were also interested in some more general questions concerning the conduct of the sessions, the amount and kind of participant involvement, and how much of the materials were covered and in what order.

During these school visits, we acted as participant-observers, only taking part in the discussion at the Leaders' discretion. Our preference was not to have a major say and, to the extent that it was possible to do so, we tended to keep in the background. We were frequently asked for the rationale for some of MART's positions and about some of the recommended teaching strategies. Our response was the same as that conveyed during the training: namely, that any and all ways of teaching mathematics was effective, provided that it didn't create or reinforce feelings of anxiety about and dislike for math, and that it didn't do an injustice to the discipline itself or to the style of the instructor. We also indicated that because interest in math anxiety was relatively new, there were as yet few firm research-based answers to pedagogical questions.

We prepared sufficient copies of all materials for Leaders' use and distribution to participants. Each Leader received a Guide and copy of the referenced handbook, and 15 copies of the handbook, the evaluation instruments, and all supplementary exercises and materials. They supplied writing paper, attendance sheets, and copies of any other materials they wanted to use.

The Guide was set up so that Leaders could prepare themselves for a session by advance reading of all relevant chapters in the handbook (as well as other suggested readings included in an extensive bibliography on math anxiety). They had also had, during the two-day Training Program, a walk-through of each session where we focused on objectives, theory and philosophy, and controversial issues.

To give Leaders a slight edge of confidence, participants did not receive the readings pertaining to the session (i.e., the chapters of the handbook) in advance; they were given these materials at the end of each session so that they could rate them in terms of how well they were covered in the workshop, in addition to how good, interesting, and useful they found them. Several participants suggested that they have access to the handbook material before the meetings; they felt that this procedure would generate more in-depth discussion, and perhaps more focused discussion as well. While this strategy has these acknowledged positive attributes, it does detract from the Leaders' need for expertise. It also may result in too much reliance on the materials, too much emphasis on the pros or cons of a particular point--with a corresponding loss of spontaneity and input by individuals.

Project staff did not establish guidelines for co-leading groups. Rather, we left this to each set of Leaders. What tended to happen at both sites that used co-Leaders (Onondaga and Westchester Community Colleges) was that responsibility for a session was divided, with one person taking charge of one part of the meeting. Housekeeping chores, however, were rarely divided equally; usually one Leader distributed and collected materials, took attendance, and arranged for space, scheduling, and refreshments.

What follows is a descriptive account of the 1981-82 MART program as it was instituted at each of the three colleges. It is based on staff observations and on information the Leaders' provided in formal assessments and informal communications. These descriptions are intended to illustrate the variations MART allows.

The Borough of Manhattan (BMCC) Series

BMCC planned a 10-hour, four-session MART workshop series, scheduled as follows:

Session 1, April 14 (2 hours)

Session 2, April 19 (2 hours)

Session 3, May 12 (3 hours)

Session 4, May 19 (3 hours)

Only three of these meetings were held by mid-May: April 14, May 5, and May 19. "End of term pressures," according to the Leader¹, originally prevented scheduling of a fourth session. (However, a fourth session was conducted subsequent to May 19, and in

¹Personal communication, June 2, 1982.

September (1982), an additional meeting took place.¹⁾

The initial decision to conduct an abbreviated workshop series reflected concern about exciting BMCC math faculty's interest in the program. To recruit enough participants, faculty feared they needed to recruit from the college's physics and chemistry faculty and then to expand the participant base to include math faculty from other CUNY two-year colleges. Between the time of the orientation meeting with the College and the first workshop session (which took place during the March Math Department meeting), an assessment was made of faculty, adjunct, and lab technician willingness to volunteer the minimum time the program required. Since sufficient BMCC math staff was interested in MART, we decided to limit participation to them, but to also limit the program offerings in this first round of implementation.

The meetings were scheduled for mid-day hours. The first one involved all regular full-time staff and a large number of adjuncts and technicians. From 10 to 18 faculty members attended each of the other workshop sessions, all of which were held in unused classrooms. The meetings were informal, with participants forming a large circle; several brought their lunch.

The attendees included several faculty who had expressed scepticism about the importance of math anxiety as an explanation of the poor performance of students. According to the Leader, these people, "senior faculty members who had never before been involved in any workshops other than in 'pure' mathematics" benefited greatly. They were stimulated, for the first time, to focus on the affective aspects of teaching."² For other faculty participants, the outcomes were equally beneficial--one person in particular reported that she had been thinking about leaving the College (mostly because of personal frustration with her inability to effect change in her math students), but that now--with the new slant MART provided--she has hope. She expressed the feelings of many participants, in this and the other colleges, that one of MART's outstanding successes was its fresh perspective on the learning problems of the community college student population.

As in the other colleges' workshops, discussion among participants was very lively. There was a great deal of sharing of ideas and experiences. Much more so than at the other sites, the Leader, after introducing a topic or concept, allowed the group unlimited time for it. As a result, less of the MART content was used, greater liberties were taken with the sequencing, and significantly less rigorous attention was

¹ Personal communication, September 28, 1982.

² Personal communication, September 28, 1982.

given to handouts and sample exercises. Since this Leader had been involved in the MART II project, her greater confidence with the materials enabled her to take a less directive stance with regard to them. As the elected Chair of the department, she also understood that her staff would react more positively to an approach that gave them the greatest latitude possible.

In comparison to the form implementation took at the other sites, the BMCC experience represented the most liberal interpretation of the program. Yet, the major outcomes were similar. Staff was stimulated to reexamine teaching strategies and motivated to reconsider the problems of learners. As we consistently found, participants gained a great deal from the interactive process itself. As at other colleges, there was not total acceptance of each and every concept the Leader introduced or that was contained in the readings; but invariably, these disagreements sparked the faculty dialogue that is characteristic of MART. As we have come to expect, the BMCC participants reacted extremely well to the opportunity MART provides to discuss the pedagogy of mathematics with colleagues.

BMCC is moving to a new building in the Fall of 1982. All of the energies of the College, therefore, are expended in that direction. The Leader reports, however, that "the mathematics faculty has requested that we continue to use the MART materials in the Spring 1983 and I am sure we will do so." Project staff feels very confident that MART will be a part of BMCC's future efforts--and that it will flourish there.

The Onondaga (OCC) Series

Onondaga Community College set up the following meeting schedule,

Session 1 March 31, 1982

Session 2 April 6, 1982

Session 3 April 14, 1982

Session 4 April 21, 1982

Session 5 April 28, 1982

Session 6 May 5, 1982

which was adhered to. Six formal sessions were conducted, starting at 3:45 p.m. and generally lasting for a full two hours. Attendance was good, averaging 12-13 people at each meeting; interestingly, attendance picked up as the series progressed. From the inception, however, MART attracted a loyal coterie of followers.

Participants included regular mathematics faculty, several part-time faculty, math lab tutors, and instructors from the Physics Department. The sessions tended to follow a similar format, closely resembling that suggested. First, one of the three

Leaders took attendance and collected the Rating Forms or other materials. Then, participants talked about the assigned handbook chapters. Generally, about half had read all the material distributed at the end of the preceeding session; several indicated that they hadn't yet done the reading, but intended to. One or two participants displayed an extreme interest in and familiarity with the handbook, pointing out some typos and errors in collating, as well as more substantive concerns. A few people filled out the Rating Forms rigorously, while others did so more sporadically indicating that while they had been keeping up with the reading they had not realized the Ratings were part of the "assignment."

Several participants mentioned that the readings (and Ratings) took them between three to five hours a week to complete. When this was talked about it became clear that it was not said as a complaint, but rather as an expression of interest. One person said that not only had she been reading all the material, but had been trying each and every suggestion with her classes--and with success. She realized that some of the in-class activities were best explored earlier or later in a semester but nevertheless felt it was important to address them all. In this way she would have some appreciation of their potential worth.

Many of the participants were amazed at their students' beliefs about mathematics and about how mathematics is done. Expressing some reservations about the phenomena MART describes, they reported that indeed students (and colleagues as well) often had here-to-fore undiscovered misconceptions, ideas, and very personal and private feelings about numbers--their meaning and manipulation. Those that had not explored these questions with students were astonished at their colleagues' reports.

At every session there was evidence that participants were coming to see things from different perspectives and were learning things that would be tried out in the classroom. Many people reported this directly, while others were less direct: for example, during a demonstration by the Leader, one faculty member kept asking how the Leader knew how to group the information she was gathering from her analysis of a word problem, and why she was writing down some of this information to one side of the board. At the time, it was apparent that the instructor who was asking the question was beginning to focus on automatic, standard teaching behaviors and their impact on students. Without saying the words, we expect that in the future this teacher will more carefully attend to the "how" as well as to the "what" he presents in class.

The three co-Leaders had obviously planned each session: each had specific responsibility for a portion of the content and each was extremely familiar with the

Guide. The Guide was used as a reference, although occasionally one of the Leaders would read from it. For the most part, they used the materials we provided very creatively, substituting examples and illustrations that they deemed more in line with the nature of their students and the problems they exhibited. There was a strong sense of wanting to make MART their own. Generally, most of the agenda was covered, at least in part: some topics were rushed, and some of the material did not hold participants' interest for the length of time allowed for it. As Leaders gain confidence and experience, they will be better able to make these necessary adjustments.

Sessions generally ended on time with most of the participants remaining in or around the room talking with a colleague or with a small group. Several stayed on for a few hours on the days of our visit, joining us for dinner and more talk.

OCC's MART series was closest to the written program in its organization and structure, in its adherence to the order of topics and their emphases, and in the Leaders' attempt to achieve the goals and objectives. Some of the content seemed to be less applicable to OCC than to more urban colleges: as one illustration, OCC faculty tend to be somewhat more satisfied with their working conditions--they tend to teach more advanced and less remedial mathematics for one thing. Thus, while the majority did not plan a teaching career around the community college, and do not have advanced degrees in mathematics or in math education, they feel more positive than do their downstate counterparts.

In our formal and informal conversations with the Leaders and with OCC faculty members we were told over and over again that the department had never before gotten together to discuss pedagogical issues--or other common problems ranging from the need for a math department lounge to departmental examinations. Individuals repeatedly told us and the Leaders that MART was a great stimulant and that they had gained a greater awareness of student reactions and how these are affected by what a teacher does or does not do.

As was the case at the other sites, faculty was more prone to discuss their excitement and interest in MART in private than to take a public stand. However, at OCC, enthusiasm continued to build and was manifested in increasing attendance at meetings, in the holding of a seventh session, and in the type of plans formulated for the future. The Leaders labelled the program successful and said, "The level of consciousness of Onondaga Community College's Math Department about math anxiety--its causes and cures--has increased exponentially....The majority of the...department members faithfully attended the 6-workshop series. They participated enthusiastically. They read their assignments...and filled out the

evaluation forms. They shared their opinions freely. New channels of communication were opened up. It was noted with regret that there are not many occasions for discussions with colleagues about common concerns. Perhaps something will be done to remedy this. The business-like format of department meetings and their agenda-packed time constraints do not allow for in-depth exploration of various issues that the MART design provides and provokes."¹

To work on followup plans, participants convened a seventh session during the week of June 2, 1982. The purpose of that meeting were to take "first steps...to implement a math anxiety workshop for students which might begin as early as this Fall 1982." In addition to the presence of eight faculty members from the mathematics department, a counselor also attended this session "and expressed interest in being a co-leader" of a team to work directly with math-anxious students.² "The chairperson of the Counseling Department...supports the idea 100%...." A second meeting of this group was set to "get down to the 'nitty gritty' of writing a detailed course outline."³ They promised to keep Project MART staff informed of their progress with the math clinic program and with other MART spin-offs: notably, a tutor training program and a re-entry program for women based on MART's objectives and approach to students.

The Westchester (WCC) Series

In many ways, the 1981-82 MART workshop program at Westchester Community College more nearly resembled the OCC program than the workshop series at BMCC. A full, 12-hour weekly program was planned and implemented as follows--

Session 1	April 5, 1982
Session 2	April 12, 1982
Session 3	April 19, 1982
Session 4	April 26, 1982
Session 5 and 6	May 10, 1982

with Sessions 1 through 4 of two-hours' duration and a one day, four-hour meeting devoted to the final two sessions. Sessions 1-4 took part late in the day, starting

¹Personal communication, June 9, 1982.

²Ibid.

³Ibid.

usually at 4:00 p.m., and generally ending at 6:00 p.m.--except for the people who lingered in animated discussion.

Also like OCC, the administration showed great commitment to the MART program, assigning it prime space and providing light refreshments and lunches. At WCC, however, the commitment was extended to small stipends for the Leader(s). Again, and similar to OCC, almost the entire mathematics faculty (as well as representatives from the Chemistry Department) participated, and on a regular basis.

However, there were important differences between Westchester and the other sites. First, WCC Leaders were the most skeptical about recruiting sufficient numbers of faculty to mount a program; in this respect, they were proven incorrect--there were about 10 faithful attendees and the first two sessions attracted the entire department because all faculty was "too curious about what was going on not to come."¹ Like the other MART workshops, the one at WCC attracted a wide variety of faculty. Included were male and female faculty members, tenured and untenured ones, and people fairly new to teaching as well as those who had been teaching for many years.

The second and perhaps related difference was the Leaders' own reluctance to undertake the assignment: at OCC, all three Leaders willingly volunteered, although they apparently subscribed to "safety in numbers"; at WCC the two Leaders, obviously the best choices and the clear preference of project staff, needed more encouragement and recognition for the time and effort the task entailed.

Faculty interest ran deep, although here too they rarely spoke openly of their enthusiasm. Both Leaders, however, reported that the participants were "grateful" to MART for the opportunity it provided to talk together. One by one they would inform the Leader(s) that they thought it was a great program, they "loved" it, and were thankful for it. Almost from the onset, as a result of the stimulation provided by the Faculty Opinion Survey (one of the initial workshop activities that also comprises a part of the evaluation), the faculty began to make plans to continue the work the program started. Their first effort was the formation of a committee to deal with the variability in the way departmental exams were being scored; at the conclusion of the program, as we will detail below, they were planning for the continuance of monthly MART-like meetings for the 1982-83 year.

The two Leaders shared most of the duties for the project with one having somewhat more responsibility for the actual content. The other functioned as a back-up, taking care of the housekeeping and other administrative chores. They were extremely well organized and worked well together.

¹Personal Communication.

Although both co-Leaders were familiar with the materials, they used them in a different manner: the Guide was always very much in evidence and often read from. The co-Leaders, in fact, referred to it as "our bible" and although they--and the participants--were quick to disagree with particular points, the Guide was clearly taken as "expert opinion." In fact in this and the other colleges' series, there was a tendency to defer to experts--a point that was continuously covered in the Training Program but one that needs reiteration in the materials: MART staff's experience with math anxiety notwithstanding, the only advantage one technique for teaching fractions may have over another technique for teaching fractions, for example, is its anxiety-reducing potential or its greater likelihood to present fewer subsequent problems for students.

Since few faculty at WCC teach both regular and remedial mathematics in any one semester, much of the MART math content was felt to be too elementary. The co-Leaders made a valiant and effective effort to isolate good pedagogical techniques that could be applied to fractions or to quadratic equations. One of the highlights of the workshop, during a chemistry instructor's presentation of quadratic equations with negative exponents, was the realization that if students were having problems it was because they had no clear conception of negative numbers.

However, the WCC group was clearly more interested in general classroom tactics and in attitudinal and affective issues than they were in specific ways to teach a concept or calculation. (One possible explanation is that the Leaders themselves were in strong disagreement with many of MART's particular strategies and may have introduced them somewhat less than wholeheartedly.) Unlike other groups of MART participants, they did not have to be led to these "philosophic" discussions. Of all groups, they spent the most time on math anxiety per se and were very quick to acknowledge it as a real phenomenon that was pervasive and that interfered with learning. They became increasingly sensitive to the concept and added the Leader's inadvertant use of the word "ergo" to a list of taboo words: therefore, obviously, and clearly.

The observations of the WCC workshop series provided confirmation that the written materials were too comprehensive for a 12-hour program. The co-Leaders made no attempt to cover an entire session's agenda, reordered topics, and often allowed the discussion to take its natural course; in one or two instances, project staff felt the session lost its focus as a result. The needs and interests of the participants predominated at WCC--at times to the extent that they shaped the session. The co-Leaders kept track of unfinished and/or deferred topics and gave the group the option

about whether or not they should be covered. Since the group was committed to the continuance of MART, several topics were therefore deferred. Others were dealt with in summary form as they were enumerated. The Leaders adopted a challenging, very informal style that seemed to work well with their colleagues; it served to involve them and motivate them to share their ideas. There was a great deal of participation and an awkward moment or two because of the Leaders' inexperience with the dynamics of small groups.

The majority of the participants kept up with the readings but did not want to be rushed. Their feeling was it was more important to read them carefully than speedily--and they intended to complete them after the series.

The last part of the final meeting was devoted to where WCC will go from here. The participants were willing to explore a variety of directions: a continuation of MART meetings for math faculty; a MART-like program targeted specifically for adjuncts and new instructors; a joint series of meetings with faculty from the technologies; and a program for secondary school teachers from feeder high schools. The group was most in favor of continuing MART discussions, but on a monthly basis starting in the Fall of 1982.

They strongly felt they should build on the sense of continuity and purpose that was developed and that they should schedule regular meetings no matter how inconvenient. They wanted more talk about math anxiety, what they do in their classrooms, and their unique experiences. They also wanted to work on difficult math problems together. Several people pointed to the session on fractions, wishing there had been more material of a similar nature. When the Leaders said there was, the group was satisfied.

Before the end of the session, responsibility for arranging the first two monthly meetings was turned over to a member of the group. He was charged with setting a schedule and with designing the content. The 1981-82 Leaders would share their materials and expertise, but future responsibility would rotate among participants.

At the end of the day-long, final session almost everyone stayed on. One person said that this "was the first time in my 13 years of teaching that I got together with colleagues on more than a one-to-one basis." Another person noted that this "was the first time that math department people got together to discuss pedagogy." A third added, "I found out useful and helpful things about my colleagues and how they conduct themselves professionally. It'll make teaching students who go on to their classes easier."

MART ASSESSMENT

General Observations

As we saw, participants felt that the MART workshop series was an excellent stimulus for bringing faculty together in a way that gave them the opportunity to get to know one another, share experiences and insights about teaching and students, and to work on common concerns. On a 4-point scale (where 4.0 = "very useful"), MART's "usefulness" was rated as 3.6. Their ratings also indicated that they learned useful things about math anxiety, and without exception, all participants said that as a result of the experience they had at least one new technique to try out with students: a new understanding of student behavior that was based on student dislike for or avoidance of mathematics; a revised way of looking at the difficulties students were experiencing; and new views about their own expectations and how these were formed. They were also more specific: one will completely change his/her way of teaching common denominators, while another said she/he will change the sequence of teaching the arithmetic of signed numbers; some said they would experiment with different forms of testing--mainly using untimed tests; others were willing to consider forms of mastery learning; and still other participants indicated they would spend more time on "review of the basics." Several participants talked about having been made more aware of how to arouse and motivate each student, and how to help each feel confident. Many talked about having in-class discussions about the importance of math, test-taking strategies, and how to read a math text. One of the participants was stimulated to consider a whole new approach to the first day of class, while others described a renewed interest in how they introduced new topics.

There were positive outcomes for groups of faculty as well: the formation of committees to work on grading and on departmental exams; the establishment of programs for special types of learners; and the setting up of math anxiety clinics and tutor training programs.

Responses to the end-of-program evaluation, "Faculty Follow-Up Survey" (see Appendix B), were highly favorable. On a 4-point scale (where a score of 4.0 is the highest rating possible, equivalent to "very"; 3.0 = "somewhat"), MART workshop participants indicated that the series was more than "somewhat worthwhile" (average score, 3.2); and between "somewhat" and "very" relevant (3.5), "interesting" (3.5), and "thought-provoking" (3.4). Participants generally liked the number of sessions and

their length, although they felt there was too little time--not enough sessions. They thought the Leaders were outstanding and that the program was well-planned and organized. They recognized and appreciated the administration's support of the program. All groups were willing to continue to meet informally after the academic year, and project staff feels that it is likely that they will continue to do so--at least for a time: once the momentum provided by external support ceases, there will probably be a falling off of regularly scheduled meetings, but these effects will be offset to some extent by the formation of new collegial relationships.

Participants also expressed some reservations about the MART program. A sizable proportion felt that they were given too much to read in too short an interval. While, as we have noted, a great many people read three to four chapters of the handbook between each session, many did not do so--although all of them indicated that they intended to read all the material before the summer holiday. There was also the suggestion that the materials be given to them in advance of the sessions and--as described previously--project staff is uncertain about whether this should be done. The point may be moot, of course, when the revised handbook is available in its entirety.

Before describing participants' reactions to the materials, we will summarize some general observations about the 1981-82 program.

To begin with, the workshop sessions at each site far exceeded our expectations and those of the Leaders. Larger numbers of faculty were recruited than were expected in each instance and they attended the meetings regularly. For the most part, they came with open minds and displayed relatively little resistance. Although all had their own ways of testing, teaching, and dealing with difficult learners, they were willing to explore new approaches to these issues--both in the workshops and in their classrooms. At least one person at each college felt that MART was the answer, more than a partial answer, to the problems and frustrations faced by instructors. These people in particular couldn't wait to try things with their class and brought back confirmation of positive outcomes. As one person said, "thanks for giving us new ways to help our students."

What was also remarkable was the development of the "team" spirit. People who had worked together, sometimes for very many years, made friendships and developed respect for and interest in their colleagues. They began to like one another and to learn from one another. In two of the workshop series there was a very heated debate--in one case about how one to grade a question and in the other about an individual instructor's way of teaching fractions. Both these incidents precipitated

discomfort. The group itself ameliorated the tension and was the primary factor in helping the participants come up with solutions. As one participant who was involved in one of these incidents reported on the Follow-Up Survey, "They [the Leaders] were very understanding of the feelings of each participant making sure that everyone felt at ease to express their ideas and not worry about what others would think. This made the group respond sympathetically and evoked us all to feel comfortable. As a result, I presented my own ideas even though they were contrary to my colleagues' ideas."

Leaders did a creditable job. When there was more than one Leader, their respective duties were well planned and carried out. Having two (or more) Leaders allows each individual to work on content he/she feels most comfortable with; it also helps vary the pace of the session and, of course, provides each an additional support. Also, the interaction between the two Leaders adds interest, just as did the dual Leadership model used in the Training Program. It is quite clear that future MART workshops would be enormously benefited by adopting a co-leader format.

It was also apparent that there was too much content to cover adequately in six, two-hour sessions. Either the number of topics needs to be greatly reduced or alternatively, the number of sessions expanded. Project staff tends to believe, as we did in our original planning of the series, that at least two more sessions would be desirable. An eight-session, 16-hour workshop series would be optimal.

However, there are advantages to maintaining the six-session format and to not reducing the scope of the content. These advantages were serendipitous. Having a backlog of topics not covered, or not covered in depth, provides participants with a repertoire of material for future meetings. For this reason, we recommend that the agenda remains intact.

Leaders need practice in dealing with the material--when to permit the discussion of a topic to continue and when to help move the group to another topic that may have more relevance. With experience they will become more adept at when to cut short discussions that are not progressing. Greater experience will also enable them to better focus the group's attention on the key points.

We have conducted MART workshops with "professional" Leaders (MART I) as well as with peer Leaders (MART III). Both techniques have advantages and disadvantages. In the former, more targeted discussions result, but it is clear that the material "belongs" to the outsider-Leaders and thus there is less sense of identification and somewhat less likelihood of lasting impact resulting from group follow-through. On the other hand, peer Leaders are less familiar with MART's underlying principles and thereby frequently lose the major thread or thrust.

Project staff believes that, overall, MART benefits from peer leadership. An extended training program would help leaders more quickly develop needed skills. However, the program profits from outsider's support and advice. Based on several years' experience with the material and with mathematics instructors, ideally MART should be comprised of two days of Leader Training, conducted by experts, followed by an additional day of training after the Leaders have conducted one or two sessions on their own.

In the following section we will summarize participants' reactions to the written materials (primarily the handbook). These data were collected to revise the handbook and are included here to provide some clues about the topics and/or issues that have the greatest interest, worth, and usefulness to mathematics instructors at two-year colleges.

Reactions to the MART Materials

Along with each chapter, each participant received a "Materials Rating Form"; the same form was used for each of the 14 chapters that comprised the 1980-81 version of the handbook. Using the same rating scale, where 1.0 = "not at all" to 4.0 which equalled "very much," participants were requested to rate each chapter on: (a) how interesting it was overall; (b) how relevant it was to the needs of students; (c) how much it stimulated thinking; (d) how well it paralleled their experiences; (e) how many suggestions it contained for new in-class activities; (f) how much it stimulated them to consider new approaches; (g) how much it prompted a re-examination of teaching styles; (h) how much it increased awareness of the needs/behaviors of math anxious students; (i) how appropriate it was to the community college math instructor; (j) how much on target it was; (k) how engaging it was; (l) how well-written it was; (m) how thoroughly the topic(s) were covered in the workshop; (n) how much workshop discussion the material generated; (o) how controversial was the material; and (p) how worthwhile was the chapter overall.

The table on page 38 summarizes their ratings. Mean scores are presented, based on the number of people who responded to an item. Responses of participants from all three colleges are combined. (Note that the N in each cell varies from 9 to 36, reflecting the number of persons who completed the form. Generally, more people completed ratings for the chapters assigned earlier than for those which formed the content of the later workshop sessions; clearly as time went on and the end of the semester approached, fewer people kept up with the assignment.)

Table 1 presents the means for each of the 16 items for each chapter, with the chapters arranged in the order they were numbered.¹ To understand the ratings, we have included an abbreviated overview of the content of each chapter. The numbers in brackets following the descriptions are references to the reordering and condensation of the materials based on the 1981-82 experience. The session number at which the chapter was discussed and subsequently distributed to participants is also noted.

Chapter 1 presents the varied, but common classroom manifestations that math anxiety takes. It introduces the reader to the phenomenon by describing symptoms math-anxious students exhibit and details many of their (mis)beliefs about mathematics. [Chapter 1.] Session #1.

Chapter 2 consists of a more formal review of the literature relating to math anxiety-avoidance and traces the career-related implications stemming from students' dislike of mathematics. The summary review starts with the early 1950's. [Literature review, technical appendix; Chapter 2.] Session #1.

Chapter 3 depicts the "anatomy" of math anxiety--how prevalent it is among community college students; how it develops and is exacerbated; and its cyclical, self-reinforcing nature. This new conceptualization permits the design of intervention strategies: preventive and ameliorative ones. [Chapter 2.] Session #1.

Chapter 4 examines how mathematicians are viewed by the public, how math competent people come to perceive themselves, and how these images adversely affect the teaching-learning environment. [Condensed; Chapter 3.] Session #2.

Chapter 5 focuses on pedagogy: here, differences in teacher-student expectations are introduced, and comparisons made between today's typical student and the college instructor-as-student. Techniques that might be effective in the classroom are introduced. [Chapter 3; Chapter 4.] Session #2.

Chapter 6 is about the first day of class. It explores practices for establishing an appropriate tone for the entire semester, by setting forth a series of questions for the instructor. Answers in the form of specific techniques are enumerated and described in detail. [Chapter 4.] Session #3.

Chapter 7 tackles word problems--the *bête noir* of mathematics. Using the Direct Translation Method of teaching word problems, instructors are presented with a strategy that engages student interest, demystifies problems, and provides students with a series of tasks that involves them in the solution. [Chapter 5.] Session #1.

Chapter 8 discusses calculus. Of import is the chapter's emphasis on the conceptual leaps that mathematics requires and on the analogies that can be drawn between the concept of limits and the algorithms for signed numbers. [Chapter 6.] Session #3.

¹1980-81 revision of the handbook.

Chapter 9 talks about fractions--the first major stumbling block for most math anxious people. Alternate ways of teaching fractions are presented, together with their respective advantages and disadvantages. Common conceptual problems in teaching and learning "good" and "bad" fractions are discussed. [Chapter 7.] Session #3.

Chapter 10 points out some of the sources of confusion in learning algebra. Several suggestions are offered that have proven worth in overcoming students' resistance to this new language. [Chapter 8.] Session #6.

Chapter 11 looks at signed numbers. The underlying concepts are examined from the learner's perspective and a new sequence of teaching is presented: addition, multiplication, division, and subtraction. [Chapter 9.] Session #4.

Chapter 12 is comprised of a series of topics recommended for in-class discussion with students. Time initially spent in clearing up students' misconceptions will be saved subsequently; talking to students also serves the purpose of humanizing mathematics and mathematicians. [Chapter 10.] Session #4.

Chapter 13 is about testing. The different purposes of tests are examined and their various forms compared and contrasted. Special attention is given to the problems of test-anxious students and how this anxiety relates to and often obscures math anxiety. [Chapter 11.] Session #6.

Chapter 14 lists current programs operative in colleges and universities in New York State that are targeted at the general math-anxiety population and at special sub-groups. Suggestions are included for setting up math anxiety clinics, courses, and other types of programs to improve students' performance in mathematics. [Chapter 12.] Session #6.

A review of the scores summarized in Table 1 on page 38 reveals that, overall (item #p), participants found the material between "somewhat" and "very much" worthwhile. Particularly worthwhile, in descending order, were Chapters 13, 7, 3, 8, 12, and 14. Of relatively least worth was Chapter 4, followed by Chapters 11 and 9.

With the exceptions of Chapters 12, 6, and 13, participants generally found the content not to be highly controversial (item #o); most topics were rated much less than "somewhat" so--especially the material in Chapters 8 and 10. They did find the reading to be much more than "somewhat" interesting (item #a), again with the exceptions of Chapters 8 and 10--and generally "relevant" to the needs of their students (item #b).

Typically, the materials were rated very appropriate for community college instructors (item #i) and fairly engaging (item #k) and well-written (item #l). There were exceptions and some wide-ranging means on a chapter-by-chapter basis, as can be seen in a detailed examination of the table.

Participants' comments provide some additional clues about the impact of the

TABLE 1

FACULTY PARTICIPANTS' RATINGS^a OF
THE MART HANDBOOK MATERIALS

ITEMS	CHAPTERS ^b													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
a. How interesting?	3.3	3.4	3.6	3.0	3.4	3.8	3.8	2.7	3.3	2.5	3.0	3.4	4.0	3.0
b. How relevant?	3.0	3.2	3.1	2.7	3.4	3.6	3.7	3.3	3.0	3.0	2.8	3.4	4.0	3.2
c. How stimulating to thinking?	3.1	3.0	3.2	2.7	3.6	3.8	3.8	2.3	3.3	2.5	3.0	3.2	3.3	3.7
d. How parallel?	3.3	3.2	3.0	3.0	3.0	3.4	3.2	3.7	2.8	3.5	3.0	3.8	3.7	3.3
e. How suggestive of new approaches?	2.0	2.2	2.5	2.0	3.2	3.8	3.3	3.3	2.5	2.3	2.0	3.2	3.7	3.8
f. How stimulating to action?	2.9	2.5	2.8	2.0	3.2	3.6	3.3	3.3	2.2	2.8	2.3	3.0	3.3	3.0
g. How stimulating to re-examination?	2.8	1.8	2.5	2.0	3.2	3.6	3.2	3.3	2.8	2.3	2.5	3.0	3.7	2.8
h. Increased awareness?	3.4	3.1	3.0	2.2	3.6	3.4	3.3	3.3	3.0	2.3	2.8	3.4	3.7	3.3
i. How appropriate?	3.7	3.5	3.7	3.5	3.4	3.6	3.7	4.0	3.3	3.3	3.3	3.4	4.0	3.7
j. How well on target?	3.5	3.0	3.0	2.8	3.3	3.4	3.3	3.0	3.5	3.7	3.3	3.3	3.7	---
k. How engaging?	3.4	3.2	3.6	3.0	3.8	3.6	3.3	2.7	3.0	3.0	2.8	3.5	3.7	---
l. How well written?	3.3	3.0	3.4	2.7	3.6	3.8	3.5	3.0	3.3	3.5	3.3	3.8	3.7	3.0
m. How thoroughly covered?	3.5	3.0	2.5	3.2	2.5	3.0	3.6	---	2.7	---	2.0	3.0	3.0	---
n. How much discussion?	3.0	3.0	2.3	3.0	2.7	3.3	3.4	---	2.7	---	2.0	2.7	3.0	---
o. How controversial?	2.4	2.3	2.5	2.3	2.2	3.2	2.6	1.5	2.7	2.0	2.3	3.6	3.0	2.7
p. How worthwhile overall?	3.1	3.2	3.5	2.7	3.2	3.4	3.7	3.5	3.0	3.0	2.8	3.5	4.0	3.5

^aScale: 1.0 = Not at all; 2.0 = Not very much; 3.0 = Somewhat; 4.0 = Very much.^bThe Chapter numbers reflect the 1980-81 ordering of the materials.

handbook. To begin with, there were many instances where they shared the material with colleagues, friends, and even family members who were not part of the MART program. One participant discussed math anxiety with his math-anxious girlfriend. Another person explored some of the concepts with a friend who had a background in education. Several spoke directly with students about particular points.

Other comments were interesting for a variety of reasons. For example, one person said (in relation to Chapter 5) the he/she "loved the many suggested techniques and approaches," but "now I have anxiety: how will I remember all this on my feet in front of my class; can I pull it off?" Another participant observed (again in relation to Chapter 5) that she/he "did most of them [i.e., the specific techniques] the next day and will try the rest."

While some people commented that much of the material was not "new," it did help put things into perspective. As one person said (about Chapter 8), "if you had not used this approach before, and if you do have difficulty teaching limits, the material is very good. But for those of us who have refined this approach, this unit [i.e., chapter, discussion] is at best a confirmation that what we are doing is right."

As expected, there were negative reactions to the materials, but not consistent ones on a chapter-by-chapter or topic-by-topic basis. For example, some people were relatively unenthusiastic about the "psychology" of math anxiety, while for others this was one of the strong topics. As another illustration, some participants felt that with respect to signed numbers, our use of the words "plus" and "minus" were okay, while others were insistent that only "positive" and "negative" be used. The question of the formality of language that should be used in teaching-learning mathematics generated a lot of discussion: at one extreme were instructors who talked about "marrying fractions," while the other extreme was represented by teachers who felt that from the outset students had to learn and use traditional terms and phrases, despite any confusion they may cause.

Participants' responses to the written materials, as recorded on the Rating Forms and by observers of the workshops, provided the MART developers with many specific ideas that have been incorporated into the most current revision of the handbook. Several chapters are omitted or moved to appendices; others have been greatly consolidated. The focus has been changed to play down the original (i.e., the CUNY) emphasis since the MART III workshop series revealed that there are significantly fewer poorly-prepared students; instructors generally do not teach both remedial and regular classes in the same semester; and there are typically fewer members of mathematics departments with doctoral degrees in colleges around New

York State than in CUNY. Perhaps related to all these factors, the expectations and frustrations of community college math instructors may not be at the level, severity, or scope that the handbook originally reflected.

Thus, while the slant of the 1980-81 version of the handbook is being moderated to take into account corrected assumptions about teachers, teaching, and learning mathematics in the community college--and more and better examples and illustrations are being incorporated--the basic document served its purposes. The MART handbook, as it was used in and as the basis for the workshop series, is:

- informative and worthwhile;
- relevant to the target audience of instructors and the needs of their students;
- interesting and non-controversial, yet provoking and stimulating;
- comprised of new ideas and concepts while reinforcing of traditional good pedagogy; and
- an inducement to self-examination, to the consideration of new approaches, and to the try-out of new strategies.

How, if at all, do instructors' generally positive feelings about the MART workshop and handbook affect the widespread and high level of math anxiety that characterizes the community college student population? The following section addresses this question.

Impact on Faculty

The crucial question to be asked about any educational innovation is whether it helps the student. Does a new program, curriculum, revised calendar plan, different grading system affect learning in a positive way or induce a climate that makes learning easier, faster, or more attainable?

The ultimate beneficiary of the MART program is the student. The rationale for MART has not changed during the course of its three-year developmental sequence: MART was and is about students and their dislike for and negative attitudes toward the study of mathematics. To keep open and even expand one's options, especially vocationally-related options, in today's and tomorrow's world means at minimal, an ease with mathematical concepts and operations. Avoiding math not only has personal implications for the student, but may have implications for the collective society. Project MART is but one of several attempts in recent years to make mathematics more attractive to more students.

Because of our premise that much of the math anxiety we witness has its etiology in the way math has been taught in the early years, MART works through the classroom instructor to bring about changes in the environment that improve the conditions for learning. In this conceptualization, the first question becomes "does MART influence teaching behavior?" (The second question, beyond the scope of the context of the MART project, but not beyond our interest, is if indeed MART does modify teaching behaviors, do the new behaviors, in turn, affect student learning?)

We have not followed MART into the classrooms of participants. On the one hand, we have faculty opinion that MART's strategies are effective and also that the phenomena MART describes are recognizable in actual classroom situations. On the other hand, we have no measure of how often any specific strategy is used by instructors exposed to MART, and no measure of how many people used its suggested techniques. But to bring about change in the behavior of teachers, it is first necessary to provide them with information, to encourage them to self-examination, and to offer them alternative ways of dealing with or justification for what they are already doing. In other words, changed attitudes are a necessary precursor to changed behavior. Do the data indicate any effects of the MART experience (workshop participation and the written materials) on the attitudes, beliefs, and expectations of college faculty, other than what they themselves report?

At the start of each workshop series, all participants were asked to complete a "Faculty Opinion Survey."¹ This instrument was designed in 1979-80 for two basic purposes: (a) to provide benchmark scores and other data for the evaluation of the MART workshop program and, (b) as a first-MART activity, to help introduce the program to participants, enable them to obtain an overview of some of the issues that would be covered, and provide them with some evidence concerning differences in how they view and react to the teaching of mathematics. (From the point of view of institutions desiring to replicate the program, the last reason is of primary importance.)

The "Faculty Opinion Survey" consists of demographic questions and attitudinal items--the latter set of questions are repeated on the "Faculty Opinion Followup

¹ A copy of this instrument is included in the Leader's Guide, Appendix A, page A10.

Survey"¹ administered at the completion of the last workshop. (We used these same instruments in MART III, thereby enabling comparisons between the MART I workshops and the current ones.)

The following results are based on the responses of 26 people (all workshop series combined) who completed both administrations of the questionnaire this year. As we will see, there were some changes from the beginning to the end of the program, indicating that participants came to view selected aspects of the instructional process and instructional role differently.

General attitudes toward mathematics. One set of 13 items dealt with some general attitudes toward math and math instruction; some of the math myths² were included. Respondents were asked to indicate the degree of their agreement or disagreement with these items in statement form. Table 2 on the next page presents their pre- and post-program scores (the lower the score, the stronger the disagreement), and the mean differences and significance, if any, between them: a plus (+) sign denotes greater agreement on the posttest than on the pretest, and a minus (-) sign shows a shift toward great disagreement at the end of the program.

As can be seen, there were changes in level of agreement-disagreement, many in the direction that reflects MART's viewpoint (items a, b, d, e, f, g, h, i, j, k, m), and significant changes on four items; on most items, however, there was no significant change. An examination of these scores generally indicates a high level of initial disagreement.

Being an effective math teacher. Faculty rated 11 items in terms of their importance in being an effective teacher of mathematics. Initial and postprogram mean scores, and differences between them (with levels of statistical significance), are shown in Table 3 (page 44): a higher score denotes greater importance, and a positive difference shows an increase in the degree of importance accorded an item after the workshop series. For most items, the desired direction of change would be to become "more important," and this obtained on all items except for c, d, and g.

¹See Appendix B. Note that the end-of-program questionnaire is also comprised of general questions asking participants to rate and describe their workshop experience. It is not an integral part of the workshop and was administered this year only for evaluative purposes.

²See Appendix A, pages A45 through A49.

TABLE 2

GENERAL FEELINGS ABOUT MATH: HOW STRONGLY PARTICIPANTS AGREE OR DISAGREE WITH 13 STATEMENTS AT THE BEGINNING AND END OF THE PROGRAM

ITEMS:	Pre	Post	Diff. (Post-Pre)
(a) Generally there is only one good way to solve a math problem	1.56	1.55	-0.01
(b) Instructors should tell students the difficulties they themselves have in math	2.81	2.91	+0.10
(c) Math is a subject that one either knows or doesn't know	1.60	1.82	+0.22**
(d) A good mathematical solution is elegant	2.60	2.64	+0.04
(e) Math is nice because it permits one to evaluate his/her own accomplishments	2.40	2.60	+0.20
(f) Math is neat and tidy	2.60	2.55	-0.05
(g) It is important to present single, clear unambiguous solutions to math problems	3.07	2.91	-0.16
(h) Calculators should not be used in class	1.73	1.82	+0.09
(i) Students should keep working on a problem until they find the solution	2.13	1.82	-0.31**
(j) Elementary school pupils should be discouraged from counting on their fingers	1.94	1.73	-0.21
(k) It doesn't matter how they do it as long as students get the right answer	1.63	2.27	+0.64*
(l) Math is not creative	1.19	1.27	+0.08
(m) Some people have a "math mind" and some don't	2.00	1.82	-0.18**
* significant at the .01 level of confidence			
** significant at the .05 level of confidence			

TABLE 3

BEING AN EFFECTIVE MATH TEACHER: PARTICIPANTS' RATINGS OF THE IMPORTANCE OF 11 FACTORS AT THE START AND END OF THE PROGRAM

ITEMS:	Pre	Post	Diff. (Post-Pre)
(a) Knowledge of math history	2.75	2.91	+0.16
(b) Knowledge of math folklore	2.50	2.63	+0.13
(c) Arithmetic skill (calculations)	3.56	3.36	-0.20
(d) Pedagogical technique	3.68	3.55	-0.13
(e) Knowledge of the psychology of learning	3.31	3.36	+0.05
(f) Preference for the specific subject	3.13	3.27	+0.14
(g) Competence in the specific subject	3.94	3.73	-0.21
(h) Knowledge of mathematical research issues	2.50	2.64	+0.14
(i) Membership in professional organizations and associations	2.44	2.55	+0.11
(j) Knowledge of student's feelings	3.56	3.64	+0.08
(k) Knowledge of how math relates to other subject areas	3.50	3.64	+0.14

Although the change in eight of the 11 items was in the desired direction, the magnitude of the changes did not reach statistical significance in any instance. The surprising items, which decreased slightly in importance, concerned the importance of pedagogical technique (item d) and the instructor's own competency and proficiency in a specific subject (item g).

Generally, however, both at the beginning and end of the program, participants felt that the same attributes are important in being an effective math instructor, and in relatively the same order of priority. The workshop program did not appear to affect their opinions in this regard.

Solving difficult math problems. Ratings of importance were also obtained for eight factors that may contribute to the solution of difficult mathematics problems. Here again we expected greater importance (higher mean scores) being attributed to each item as a result of participation in the program.

Table 4 on the following page summarizes participants' scores on the two administrations of the questionnaire and the mean difference between them. As can be seen, on half the items, the scores changed significantly: practice, knowing how others did the problem, and knowing what has previously failed, all increased in

importance; arithmetic ability became less important. Disappointingly, intuition tended to be rated slightly less important also, but not significantly so. (MART stresses the role of intuition, perserverance, and practice, and deemphasizes logic and arithmetic ability in the solving of problems.) These results are equivocal, suggesting that participation had some desired impact in the hoped-for direction.

TABLE 4

SOLVING DIFFICULT MATH PROBLEMS: PARTICIPANTS' RATINGS OF THE IMPORTANCE OF EIGHT FACTORS AT THE START AND END OF THE PROGRAM

ITEMS:	Pre	Post	Diff. (Post-Pre)
(a) Ability to do arithmetic calculations	3.25	3.00	-0.25
(b) Knowing basic mathematical concepts	3.94	3.73	-0.21
(c) Knowing how others did the problem	2.53	2.90	+0.37
(d) Knowing what has failed in previous attempted solutions	3.07	3.36	+0.29
(e) Logic	3.44	3.27	-0.17
(f) Intuition or guessing	3.27	3.18	-0.09
(g) Perserverance	3.56	3.64	+0.08
(h) Practicing similar, but easier problems	3.31	3.72	+0.41

Assigning course grades. Another set of 10 items dealt with factors that may be involved in assigning grades to students. Here we made no predictions about the desired direction of change. Table 5 below presents the mean scores for each item on the pre- and postprogram administration of the Faculty Opinion Survey. Note that the higher the mean score, the more important is the factor.

TABLE 5
ASSIGNING COURSE GRADES: THE IMPORTANCE OF 10 ITEMS
AT THE START AND END OF THE PROGRAM

ITEMS:	Pre	Post	Diff. (Post-Pre)
(a) Performance on the final	3.56	3.64	+0.08
(b) Performance on the mid-term	3.31	3.45	+0.14
(c) Performance on quizzes	3.13	3.00	-0.13
(d) Participation in class	2.43	2.64	+0.21
(e) Completion of homework	2.57	3.09	+0.52
(f) Student's general ability level	2.29	2.64	+0.35
(g) The effort the student makes	3.13	3.18	+0.05
(h) The student's general attitude	2.80	2.91	+0.11
(i) Your general impression	2.07	2.64	+0.57
(j) Departmental policy	2.33	2.70	+0.37

An examination of the ratings in Table 5 indicates that with one exception (item c), each factor became more important in determining grades: the exception was performance on quizzes, and this was in the desired direction. Note also that the change in three items was significant--as a result of MART, participants more strongly believed in the importance of the instructors' general impression (item i), completion of homework (item e), and departmental policy (item j). The first two are understandable in terms of the program's thrust, but the significant change in the last item is inexplicable other than perhaps reflecting the groups' increasing awareness of the great variability in grading that exists among the instructional staff.

Partial credit. This variability is seen most dramatically in three solutions to each of two problems--algebraic problems--that respondents were asked to grade. The respondent was to assign a grade of between 0 (no credit) to 10 (full credit) to each of the three incorrect solutions that was presented. The first solution to the first Problem (A) was a trial-and-error approach that involved no algebraic manipulation. The other two solutions represented, respectively, carelessness and a transpositional error that nevertheless showed some understanding of algebra.

Participants' mean scores to the first problem are presented in Table 6 below. The pre-postprogram differences are highly significant: there was an increase in the amount of partial credit awarded to each incorrect solution--and of more than 2 points to the first one. The scores ranged from 0 to 10 on solution #1, and from 2 to 9 on solutions #2 and #3.

TABLE 6

AWARDING PARTIAL CREDIT: MEAN GRADES PARTICIPANTS ASSIGNED TO
3 INCORRECT SOLUTIONS TO 2 PROBLEMS AT THE START AND
END OF THE PROGRAM

<u>Problem A</u>		Pre	Post	Diff.
Solution	1.	1.00	3.09	+2.09
	2.	5.63	5.82	+0.19
	3.	7.00	7.27	+0.27
<u>Problem B</u>		Pre	Post	Diff.
Solution	1.	4.33	3.73	-0.60
	2.	6.20	5.36	-0.84
	3.	8.00	7.18	-0.82

Similar significant changes occurred with respect to the second problem (B). Again, the task was to grade three incorrect solutions to the problem. Here the first solution contained multiple errors that could be attributed to carelessness, anxiety, or a lack of understanding of how to handle equations involving fractions. The second incorrect solution had two sloppy errors, and the third involved one obviously careless mistake involving a sign in an otherwise knowledgeable solution. Table 6 contains participants' scores.

On the second problem (Table 6, page 47), participants became significantly less tolerant of incorrect answers. There was a decrease of from .6 to .8 points in partial credit--partly because initially a large amount of credit had been awarded in each instance. There is also the suggestion that they became less lenient about errors due to carelessness and much more tolerant about inelegant attempts to solve problems.

Overall, the pre-postprogram differences on the Faculty Opinion Survey and Followup are not dramatic and not nearly as striking as those we found in the MART I series using similar measures.¹ The explanation may lie in the difference in emphases between the MART I and MART III workshop programs. In MART I, where project staff (including consultants) acted as leaders, much more stress was placed on the factors that relate to the items under discussion. As we have pointed out, the current series--led by trained peer leaders--was less focused around these issues.

Do these questionnaire results mean that there were fewer attitudinal changes (and hence, fewer behavioral ones) as a result of MART III program participation? Not necessarily. The observations suggest that the questionnaire items measured aspects of feelings and beliefs that were not as well-addressed by the MART III program as by the series conducted in the project's first year. This is borne out by participants' reports. The instrument was not revised to reflect the project's growth and was, therefore, not necessarily picking up the changes that were occurring. Faculty indicated by word and deed throughout the program that they were looking at the learning and teaching of mathematics in innovative ways and, more importantly, were adopting some of the recommended strategies and techniques.

When asked directly whether MART changed their ideas or stimulated them to new considerations, they rated the program "very helpful." It was most helpful in leading them to reexamine their teaching techniques, students' reactions to mathematics, and the needs of the math anxious student; MART was rated somewhat less helpful at increasing their understanding of their own expectations for students, and at how much participants themselves take for granted as mathematicians. The program was judged relatively least helpful--but still greater than "somewhat"--in changing the way faculty taught fractions, signed numbers, and algebra. However, in the words of one participant "neither the rating forms nor the Survey do justice to our experience. There wasn't the time nor the space for us to adequately describe what we truly got out of the program."

¹Heller, et al., 1980. op. cit.

SUMMARY AND CONCLUSIONS

During the 1981-82 academic year, with a third and final grant from the New York State Education Department, the CASE Institute for Research and Development in Occupational Education implemented a model staff development workshop series. Designed for community college math faculty and representatives of other math-based disciplines, the current MART (Mathematics Anxiety Reduction Training) project--which was begun in 1979--sought to improve student performance in mathematics, with a resultant expansion of their career options, by focusing on the classroom learning environment.

The research on math anxiety documents its pervasiveness and severity among large portions of the population, including students enrolled at the community college. If the phenomenon is indeed as widespread as supposed and if further research supports the hypothesis that math anxiety has its genesis in classroom experiences, then the classroom itself might be a more appropriate site of redress than the out-of-class math clinics proliferating around the nation.

MART was initiated in response to these assumptions. By working with the instructor to change the conditions of learning, we anticipated a reduction in students' anxiety-avoidance reactions; this reduction in anxiety, in turn, would promote learning and improved performance. A program for classroom instructors was the basic premise.

MART I, the first-year program (1979-80), was comprised of two workshop series, conducted by project staff and consultants, for faculty participants. Preliminary materials were developed for participants' use. All indications were that the project was effective in recruiting appropriate faculty from the two-year colleges, involving them in an examination of pedagogical questions, and impacting positively on their attitudes toward and knowledge about math-anxious students and other students who exhibited a strong dislike for mathematics.

As a result of MART I's success, MART II was initiated. During 1980-81, project staff, with the assistance of a large number of math instructors drawn primarily from the CUNY system (but with some representation from the SUNY two-year colleges), developed a handbook of readings and supplementary exercises and activities which were field-tested for readability and usefulness.

In 1981-82, the final year of the project, staff and the project's long-time

consultant trained eight mathematics professors from five different community colleges (four SUNY and one CUNY school) in how to conduct a MART workshop series. A Leader's Guide was developed for use, which was employed in concert with the handbook that was made available to all faculty participants. Workshop series were initiated by three colleges; the other two sites intend to conduct the program during the 1982-83 academic year.

Basically, the model workshop program consisted of six, two-hour sessions involving approximately 10-15 faculty participants. Two colleges followed this format fairly closely, while the other adapted it to better fit institutional needs. At all three sites, the majority of the regular, full-time faculty were involved in the program, in addition to some adjunct faculty and lab technicians; representatives of chemistry and physics departments were also in attendance. Attendance at the meetings was excellent, and each school had a dedicated coterie of attendees who came to all sessions.

The original design called for a total of 16 workshop hours, in eight, two-hour blocks; this was modified in response to Leaders' perceptions that it was too great a commitment from faculty. Although the sessions were reduced to 12 hours to better accord with their feelings, everyone--staff, Leaders, and participants--agreed that more than 12 hours was needed. Participants appeared more than willing to have additional meetings, and at each site followup meetings were conducted, primarily for the purpose of planning activities that would build on the project's accomplishments. One college had a planning meeting to design a monthly series of workshops for the 1982-83 year; at another college, plans were put into effect to develop a tutor-training program and a math clinic for targeted populations. The third site conducted extra meetings after having initially limited the year's program to four.

The training of the Leaders took place during two days in March 1982. Because of the amount of information to be conveyed--about math anxiety, conducting peer-led workshops, and about group dynamics--the training was based on a modeling design. The project director and consultant co-led the group, taking the Leader's through the program step-by-step, at a highly accelerated pace. Although the Leaders felt the training was generally good, and the participants rated them as capable, competent leaders, it is clear that additional training time is needed. MART calls for a style of leadership very different from that of most teachers, and also asks the Leaders to transmit information and teaching techniques they might not believe in. Although we emphasized the multiplicity of "right ways to teach anything," it was often beyond the experience of the Leaders to cope with or to appropriately focus the groups'

involvement. As a result, despite the many positive outcomes, several important topics were not addressed well. Also, as a result of inexperience with the principles of group dynamics, there was less closure and less focus than hoped for.

From the participants' point of view, however, the program was outstanding--far superior to any other staff development activity they had been involved with. While they enjoyed the handbook and rated it informative and useful, they indicated that it was too much to read; one participant reported that she spent 5+ hours a week just reading the assignments.

However, she also tried each and every suggestion with her classes and explored each and every idea with students. She was not alone; almost every participant was able to describe at least one new concept or approach that he/she had tried out or were willing to try out in class. They learned from one another as well as from the Leader and materials. They learned very specific new strategies, such as new sequences for teaching computations involving signed numbers, to more general ones, such as using untimed tests. They reported they also learned about the characteristics of math-anxious students and about themselves. One man learned "patience"; others talked about being reminded of the need for continual review.

As least as important as the particular pedagogical techniques were the interpersonal outcomes: colleagues who had often worked together for years got to know each other for the first time. They were made to feel comfortable and willingly shared experiences. Clearly MART affords a unique opportunity for the growth of individual faculty and for the solidifying of a department.

MART had the active support of the administration at the three colleges that implemented the program this year. Administrative backing was evident in the allocation of space, in the provision of refreshments and, in one case, a small stipend for the Leader(s). The administration has encouraged future plans and the likelihood is that these will materialize.

In its present form, the MART program consists of a handbook in its second and final revision, a completed Leader's Guide, and a two-day Leader Training Program. MART, in one of its several developmental stages, has been effectively implemented at two CUNY Community Colleges, two SUNY Community Colleges, and there has been a series involving faculty from other CUNY two-year schools and some private schools in New York City. This totals almost 75 persons. In addition, faculty from two more SUNY two-year colleges have been trained, and almost two dozen other people have been involved with some aspect of the total program, generally with the design of the handbook. Thus, there currently exists a large number of individuals who have been exposed to MART.

Project MART has already generated a great amount of related activity: MART participants have gone on to develop other programs based on the MART philosophy, and MART "graduates" are active in math departments and in professional organizations. There is every indication that because of their interest, a classroom-based, math anxiety-reduction solution to math anxiety will continue to grow.

MART's work is not quite done:

- A final version of the handbook has to be published and made available to a wider public;

- An expanded Leader Training Program should be tested; and

- The interest of MART participants, individuals and departments, needs to be reinforced.

Finally, and of first importance, is the yet-to-be answered question of whether MART is having an effect on the attitudes and performance of the hundreds of students in classes taught by participating instructors.

APPENDIX A
LEADER'S GUIDE

Author's Note:

This Guide is intended for use by mathematics instructors wishing to conduct staff development workshops for math faculty around the issues of anxiety-avoidance in community college students.

It was developed to be self-contained--being comprised of all exercises, activities, and examples for participants as well as of all the organizational material and much of the background information workshop Leaders' require. To maximize the program's effectiveness, the author urges that leaders' familiarize themselves with the math anxiety phenomenon (and a bibliography of readings can be found in Appendix C) and the MART approach (this Project is also described in several reports). The Guide is, however, cross-referenced to the MART handbook which will be published shortly under separate cover. In the interim, institutions planning to initiate a MART workshop program may contact the author directly for draft versions of the handbook.

Two sets of references to the handbook are shown in the Guide. The first refers to the appropriate chapters in the 1980-81 version; the chapter numbers in brackets pertain to the most current, 1981-82, revision. A further modification--expansion--of these materials is anticipated. The final handbook will be available in the Spring of 1983, in adequate time for use in the fall semester.

Center for Advanced Study in Education
 Institute for Research and Development in Occupational Education
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 City University of New York

MART: Mathematics Avoidance Reduction Techniques (III)

LEADER'S GUIDE
 AN OVERVIEW

MART III is a six 2-hour workshop program developed by community college mathematics instructors for community college mathematics instructors.

As a staff development program, MART is designed to be run on your campus for your faculty but can include faculty from math-based curricula: physics, chemistry, and the technologies--as well as staff from neighboring postsecondary institutions.

The problems MART addresses are both attitudinal and mathematical. As such, MART responds to the needs of students in both remedial and regular mathematics courses and is specifically targeted at those who dislike math, experience anxiety about math, are poorly prepared to deal with math, and who are generally unable to do so.

MART's focus is on actions that the classroom teacher can take to improve student learning. The program takes participants through specific techniques concerning fractions, word problems, algebra, and the limits concept, as well as strategies for testing, creating an open environment, and setting up math clinics. However, the program's basic premise is that the richest repository of information resides in the combined experiences of you and your colleagues.

MART seeks to initiate a dialogue that taps this resource.

Center for Advanced Study in Education
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MART: Mathematics Avoidance Reduction Techniques (III)

MART WORKSHOP PROGRAM OVERVIEW

SESSION #1

What is math anxiety?
Who's got math anxiety?
My math anxiety
Word problems

SESSION #2

More about word problems
The public's image of mathematics
Mathematics: what is it?

SESSION #3

Characteristics of math anxious students
Fractions
Classroom Do's and Don'ts

SESSION #4

Math myths and their consequences
Signed Numbers
Topics for classroom discussion

SESSION #5

General teaching practices
Calculus

SESSION #6

Algebra
Testing/grading practices
Textbooks: selecting them, reading them
Where do we go from here?

Center for Advanced Study in Education
Institute for Research and Development in Occupational Education
Graduate School and University Center
City University of New York

MART: Mathematics Avoidance Reduction Techniques (III)

WORKSHOP SESSION #1

AGENDA

To prepare, read this section of the Leader's Guide
read chapters 1, 2, 3, and 7 [1,2, and 5]
read "Additional Readings"
review the Faculty Opinion Survey
review the Toy Boat Problem and solution

A. Welcome

- Registration
- Faculty Opinion Survey

B. What is math anxiety?

- a brief history
- a current working definition

C. Who's got math anxiety?

- a brief review of the research literature
- a brief discussion of how math anxiety can be recognized:
formal measures
informal assessments

D. My math anxiety

- the Toy Boat problem
- discussion of reactions and the solution

E. Word problems

- introducing the Direct Translation Method

MATERIALS FOR DISTRIBUTION TO WORKSHOP PARTICIPANTS

- MART workshop program outline (Handout #1.1)
- *the Faculty Opinion Survey (Handout #1.2)
- *the Toy Boat Problem (Handout #1.5)
- "Additional Readings" (Handout #1.6)
- MART II: Chapters 1, 2, 3, and 7 [1,2, and 5]

*For use in Session #1

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A. WELCOME (15 minutes)

Purpose: to outline the program's goals and the topics that will be covered.

As participants arrive,

- ask them to sign in.
- distribute copies of the Workshop Program Outline, #1.1. (This is a summary of the topics that will be covered in each of the 6 sessions.)
- distribute, and invite them to complete the Faculty Opinion Survey (#1.2)
- as they complete it, leader outlines MART's goals:
 1. To explore the nature and extent of math anxiety-avoidance and its specific impact on the teaching and learning of mathematics at the community college;
 2. To introduce teaching strategies that can be used in the classroom to counteract some of the deleterious effects of student math-anxiety-avoidance. (You might want to mention that MART was one of the earliest programs that sought to combat math anxiety in the classroom, and by the classroom teacher);
 3. To devise plans for on-going activities at the college.
- Leader collects and responds to questions about the Faculty Opinion Survey:
 1. It serves ice-breaking purpose--stimulates discussion and provides a common starting point;
 2. It focuses attention on many of the issues that will be discussed subsequently (e.g., sex differences in math ability, how instructors model their teaching behavior, awarding partial credit, and so on); and
 3. It should also remind participants what it feels like to be "tested" when they enter "class" for the first time.

B. WHAT IS MATH ANXIETY? (15 minutes)

Purpose: to brief participants on the history of math anxiety, introduce definitions of the phenomenon, and stimulate an awareness of how math anxiety hampers their teaching efforts.

- Important points to be covered include,
 1. Math anxiety is a real and vital phenomenon and one that has important implications for instructors as well as students;

2. It is no great discovery that people dislike math. What is significant is that negative attitudes and anxieties toward math adversely affect learning;
 3. Students' negative attitudes lead to poor performance and restrict future choices (you may want to cite Lucy Sells' study) STET
 4. Because they themselves were good in math, instructors tend to give less credence than is warranted to the importance of this phenomenon.
- In presenting a historical overview (chapter 2 [2]), mention that
 1. Teachers have always been aware that most people don't like math and that many are made anxious by it. (Discuss some personal anecdote);
 2. That it first appeared in the scholarly literature about 30 years ago (Gough, 1954) and has been reappearing under a variety of labels (mathophobia, mathephobia, number anxiety, math anxiety, math wariness)--and has recently been linked to math avoidance;
 3. That it recently has received an inordinate amount of attention, largely as a result of the Woman's Movement and of the shortage of scientists, engineers, and technicians; and
 4. That there are various definitions of math anxiety, with slightly different emphases, but they also have commonalities.
 - Introduce the working definition of math anxiety,

"Math anxiety describes a range of emotional and negative feelings about mathematics that causes its victims to suffer mild to severe physical discomfort; interferes with their ability to concentrate, pay attention, and assimilate mathematical information; and impedes their capacity to deal with mathematical concepts and manipulations in school, in their choice of career and at work, and in everyday situations."

"Math anxiety occurs in otherwise able people and is at a level of awareness that causes its victims to question their abilities. It occurs in people of all types, but is more prevalent among women and among people with poor preparation in math. It is common and pervasive and frequently associated with early traumatic experiences and with irrational (magical) beliefs about how math is done."

 1. Suggest that participants read chapters 1 [1], and 2 [2] and the "Additional Readings" handout #1.6 (which are distributed at the end of the session), and
 2. Mention that the definition presented here and the model in Chapter 3 [2] not only reflects the view that math anxiety can be reduced, but that different ameliorative strategies may be required depending on its developmental stage and severity.

C. WHO'S GOT MATH ANXIETY? (30 minutes)

Purpose: to document the prevalence of math anxiety among the general population, including community college students; and to introduce participants to formal measures and informal symptoms that suggest the presence of math anxiety.

- Briefly review the research literature, (chapter 2 [2]), highlighting that:
 1. One-third of the college students surveyed by Dreger and Aiken (1957) were found to have "number anxiety", a condition that not only differed from and was relatively independent of "general anxiety" and "test anxiety", but which was not correlated with intelligence;
 2. In 1972, Richardson and Suinn (authors of the "Mathematics Anxiety Rating Scale") found that 20% of a sample of undergraduates showed extreme tension associated with mathematics and that more than one-third of the students who volunteered for a therapy program offered by the university counseling center had problems that centered around math anxiety;
 3. Heller, D'Lugin, and Kogelman (1980) found high levels of math anxiety among urban community college students, with disproportionate numbers who were female or who were in remedial math courses;
 4. Students' math anxiety is also being addressed at Harvard University, Wesleyan University and Mills College, as well as in schools for American Indians; and
 5. Although more usual among the poorly prepared, math majors also exhibit anxiety reactions.
- OPTIONAL: Distribute the MARS and MAS, and tell participants that
 1. These two instruments are in common use today;
 2. They are essentially based on self-reports and, as such, are likely to elicit more or higher levels of anxiety from people more willing to describe their feelings (which may account for women's greater math anxiety);
 3. The two instruments should be compared and contrasted in terms of what the purport to measure.
- Discuss informal ways of assessing math anxiety;
 1. The easiest way, and an accurate one, of determining if a student has math anxiety is to ask him/her;
 2. Most people, even those we meet socially, are delighted to describe how much they dislike math, the trouble they have doing it, and how it all started (that is, what they attribute their math anxiety to-- see Chapter 1 [1]; and

3. While aware that they "have it", and that many experience physical symptoms (palpitations, sweaty palms, a black curtain), few are aware of its other implications. These implications include (see Chapter 1[1]).
 - poor attitudes toward and dislike of math.
 - inability to concentrate.
 - difficulty in learning new material.
 - belief in Math Magic and Myths.
 - feelings of stupidity and incompetence.
 - feelings of distrust about math and toward math instructors.
 - fear of "discovery"--of being found out.
 - dismissal of math as important in "my life".
4. In the classroom, math anxiety can manifest itself in a wide range of behaviors, the purpose of all of which is to reduce feelings of tension, fear, and discomfort. (Chapter 1 [1])
 - denial: students deny their anxiety by mimicking socially acceptable classroom behavior; until the first test they look like they are learning...
 - substitution: students find activities to substitute for anxiety. They take copious notes, fuss...
 - withdrawal: students appear disinterested or lackadaisical, they tune out...
 - beligerance: students project hostility and engage in antagonistic behavior; they can disrupt the class...

D. MY MATH ANXIETY (40 minutes)

Purpose: to illustrate that most people, including mathematicians, experience some level of math anxiety at one time or another (but that it is generally not debilitating); and to present participants with a situation that enables them to experience some of the frustration their students experience when faced with a problem at their level of difficulty.

- Distribute the Toy Boat Problem (#1.5), tell participants that they have 15 minutes to solve it. Ask them to keep track of their feelings as they work on it.
 1. As participants work, you may want to jot down their comments ("not enough information," "it's a trick question") or note their behavior (some will attack it, others may not).
 2. When the time is up, or when all have solved it or quit trying, ask for their reactions.
 3. Point out the similarities between how they felt and how students feel when they are given problems which they experience to be as difficult.
 4. Of course, if they want to, show them the solution. (Chapter 7[5])

5. The most important conclusion to be drawn, however, is that even math-competent and confident people experience anxiety but how it affects them is different. Because they are confident of their ability, the challenge of the problem stimulates them to work toward a solution. Because they know how math is done, they are able to try more than one approach. Typical students, however, do not react the same way. Rather than working harder, they give up. Because they believe that there is only one way to do math, they get discouraged if one way doesn't work. But students' feelings and initial reactions and participants' feelings and initial reactions are very similar.
- Discuss participants' negative experiences with math. The purpose of this discussion, as was that about the Toy Boat problem, is to stimulate new empathy for students.
 1. By remembering our own worst experiences it is possible to understand what many of our students feel a great deal of the time.
 2. You can start by relating your experiences as a student and prompt further discussion by asking questions such as
 - did anyone else have difficulty with math in school?
 - what did you do? What were your classmates doing?
 - what effect did it have on you to see others getting it while you were still lost?

E. WORD PROBLEMS (20 minutes)

Purpose: using the Toy Boat problem as a jumping-off point, to illustrate an effective way to teach the solution of word problems. (Work on word problems will be continued in WORKSHOP SESSION #2.)

- Introduce the Direct Translation Method (Chapter 7 [5]) of solving Word problems.
 1. It is an effective strategy with word problems varying in difficulty.
 2. It leads to a solution and at the same time confronts an important myth that students believe in:
 - the myth that math is done quickly, and in your head.
 3. The Direct Translation Method focuses student attention on the fact that the solution to a problem lies in clues in the problem, but that often some other information must be brought into play:
 - refer to the things other than math-related that have to be taken into account in the solution of the Toy Boat problem (e.g., upstream = against the current; downstream = with the current, "toy" boats don't have a motor, etc.).

- discuss how frequently other-than-mathematical information is necessary to solve problems and that some of the time this may be the cause of difficulties students' have (e.g., to compute an annual salary, one has to know the number of weeks or months in a year; on a room-painting problem, students have to account for the ceiling, and so on).

- Ask participants to describe how the typical student attacks word problems and how they teach them.

1. Students look at a problem, read it over and over again looking for an answer, become more and more confused and overwhelmed when they don't immediately know what to do.
2. Have participants describe how they teach word problems. You may want to point out the similarities and differences between their methods and the Direct Translation approach:
 - the problem is not read through completely
 - single words or phrases are read one at a time
 - students start working (writing) immediately
 - halfway into the problem the solution is set out

- Suggest that participants read Chapter 7 [5] for the next session.

1. Distribute copies of Chapter 7 [5].
2. Highlight the agenda on their Workshop Program Outline (#1.1), telling them they will work through some word problems next time.
3. Distribute copies of Chapters 1, and [1 & 2] 2, and the "Additional Readings" (#1.6) explaining generally that they cover some of the topics discussed in today's session.

Project MART: FACULTY OPINION SURVEY, 1981-82

This opinionnaire is designed specifically for mathematics faculty. It consists of 3 sections: your teaching preferences and experiences, beliefs and classroom styles, and your own background and experiences as a student.

The purpose of this instrument is two-fold: First, consider it a technique for elucidating some of the feelings and perceptions we hold as mathematicians; as such, it will provide us as workshop participants with a starting point--a common basis for discussing events that take place in the classroom. Second, it will provide essential information which, together with your responses to other questions we will ask periodically throughout the workshop, will enable us to evaluate MART's effectiveness.

This opinionnaire will be collected at the end of today's session; blank copies of this form are available for those who would like one. Although we guarantee anonymity and confidentiality of individuals, we need to be able to identify the same respondent to later questions. Therefore, please code your opinionnaire by using the last 4 digits of your Social Security number. We appreciate your cooperation. Thank you.

Please enter those 4 digits here: _____

SECTION I: TEACHING PREFERENCES

1. What math courses do you most prefer teaching? _____
2. What math courses do you least like to teach? _____

3. Please indicate your preference for teaching the following types of students and courses:

		Prefer to teach	Don't mind	Do not enjoy
students	Remedial students			
	Average students			
	Bright students			
	Bilingual students			
school level	Pre-high school			
	High School			
	Community college			
	Four-year college			
	Graduate school			

4. We are interested in whether, in your experience, male and female students differ with respect to their performance in mathematics and in their behavior in the classroom. For each item below, circle the M or F to indicate who generally performs the task better or is more likely to engage in the behavior.

Arithmetic.	M	F	Willing to work at the blackboard.	M	F
Fractions.	M	F	Asks questions during class.	M	F
Word problems.	M	F	Asks questions after class.	M	F
Geometry.	M	F	Completes homework assignments.	M	F
Trigonometry.	M	F	Attends class regularly.	M	F
Algebra.	M	F	Answers questions in class.	M	F
Statistics.	M	F	Is nervous in class.	M	F
Calculus.	M	F	Is nervous taking tests.	M	F

A10

SECTION II. BELIEFS AND PRACTICES

1. Listed below are a number of statements. Please rate how important you feel each is in being an effective teacher of mathematics.

	Not Important At All	Somewhat Unimportant	Somewhat Important	Very Important
Knowledge of math history				
Knowledge of math folklore				
Arithmetic skill (calculations)				
Pedagogical technique				
Knowledge of the psychology of learning				
Preference for the specific subject				
Competence in the specific subject				
Knowledge of mathematical research issues				
Membership in professional organizations and associations				
Knowledge of students' feelings				
Knowledge of how math relates to other subject areas				

2. For each of the following statements, please indicate how you feel.

	Strongly Disagree	Disagree Somewhat	Agree Somewhat	Strongly Agree
Generally there is only one good way to solve a math problem.				
Instructors should tell students the difficulties they themselves have in math.				
Math is a subject that one either knows or doesn't know.				
A good mathematical solution is elegant.				
Math is nice because it permits one to evaluate his/her own accomplishments.				
Math is neat and tidy.				
It is important to present single, clear, unambiguous solutions to math problems.				
Calculators should not be used in class.				
Students should keep working on a problem until they find the solution.				
Elementary school pupils should be discouraged from counting on their fingers.				
It doesn't matter how they do it as long as students get the right answer.				
Math is not creative.				
Some people have a "math mind" and some don't.				

3. The following factors are believed to contribute to the solution of difficult math problems. Rate each.

	Not Important At All	Somewhat Unimportant	Somewhat Important	Very Important
Ability to do arithmetic calculations				
Knowing basic mathematical concepts				
Knowing how others did the problem				
Knowing what has failed in previous attempted solutions				
Logic				
Intuition or guessing				
Perserverance				
Practicing similar, but easier problems				

4. Many factors go into computing a student's grade. Please indicate how you feel about each.

	Not Important At All	Somewhat Unimportant	Somewhat Important	Very Important
Performance on the final				
Performance on the mid-term				
Performance on quizzes				
Participation in class				
Completion of homework				
Student's general ability level				
The effort the student makes				
The student's general attitude				
Your general impression				
Departmental policy				

5. Below are two math problems similar to ones that you might include on an exam in one of your courses. For each problem three typical solutions are given, all of which have incorrect answers. The maximum credit for each problem is 10 points. Please indicate how you would grade each solution by scoring each from 0 to 10.

Problem A: BILL HAS \$5.05 IN DIMES AND QUARTERS. IF THE NUMBER OF QUARTERS EXCEEDS TWICE THE NUMBER OF DIMES BY ONE, FIND THE NUMBER OF COINS OF EACH TYPE.

Solution #1

10 quarters = \$2.50
 20 quarters = \$5.00
 18 quarters = \$4.50, 5 dimes = 50¢, total = \$5.00
 15 quarters = \$3.75, 13 dimes = \$1.30, total = \$5.05
ans. 15 quarters, 13 dimes

SCORE: _____ POINTS

Solution #2

Let no. dimes = d
 Let no. quarters = $2d-1$
 value of dimes = $10d$
 value of quarters = $25(2d-1)$
 $10d + 25(2d-1) = 505$
 $10d + 50d - 25 = 505$
 $60d = 480$
 $d = 8$
 $2d - 1 = 15$
ans. 15 quarters, 8 dimes

SCORE: _____ POINTS

Solution #3

total coins = \$5.05
 dimes = $10N$
 quarters = $25(2N+1)$
 $10N + 25(2N+1) = 505$
 $10N + 50N + 25 = 505$
 $60N = 540$
 $N = 9$
 $2N + 1 = 19$
ans. 19 quarters, 9 dimes

SCORE: _____ POINTS

Problem B: SOLVE FOR a : $-25 = \frac{7}{3}a - 11$

Solution #1

$$-25 = \frac{7}{3}a - 11$$

$$-11 \quad -11$$

$$-36 = \frac{7}{3}a$$

$$\frac{3}{7}(-36) = \frac{3}{7}a \left(\frac{3}{7}\right)$$

$$a = 5\frac{1}{7}$$

SCORE: _____ points

Solution #2

$$3 \cdot -25 = \frac{7}{3}a - 11 \cdot 3$$

$$-75 = 7a - 11$$

$$+11 \quad +11$$

$$-64 = 7a$$

$$a = -9\frac{1}{7}$$

SCORE: _____ points

Solution #3

$$-25 = \frac{7}{3}a - 11$$

$$+11 \quad +11$$

$$3 \cdot -14 = 3 \cdot \frac{7}{3}a$$

$$\frac{42}{7} = \frac{7a}{7}$$

$$a = 6$$

SCORE: _____ points

SECTION III. DEMOGRAPHIC INFORMATION

1. What was your undergraduate: Major? _____ Minor? _____

2. What is your highest earned degree?

☐ AB/BS ☐ MA/MS ☐ Ph.D./Ed.D. ☐ Other; specify: _____

In what field? _____

3. Are you currently pursuing another degree? ☐ No ☐ Yes; if yes, what degree? _____
What field? _____

4. Please list the courses you currently teach: _____

5. What kinds of students are you now teaching? ☐ Remedial ☐ Average ☐ Advanced
AND ☐ High school-aged ☐ College level ☐ Adults ☐ Other; specify: _____

6. When (at what age and/or grade level) did you first decide that you were interested in mathematics as a profession? _____

7. Please describe the feelings, event(s), or experiences underlying your personal decision to pursue a career in mathematics: _____

8. When you were in high school, how did you feel about each of the following subjects?

	Strongly Disliked	Disliked	Liked	Strongly Liked
English and literature				
Foreign Language				
Physical, chemical sciences				
Biological, natural sciences				
Mathematics				
History and the social sciences				

9. In general, how would you describe your high school math instructors?

Boring _____ Interesting _____
 Excellent _____ Poor _____
 Warm _____ Cold _____
 Indifferent _____ Concerned _____

10. In general, how would you describe your college math instructors?

Boring _____ Interesting _____
 Excellent _____ Poor _____
 Warm _____ Cold _____
 Indifferent _____ Concerned _____

11. What was the lowest grade you ever received in a math course? _____
 What course? _____

12. As a student, what math course(s) did you find most difficult? _____

13. Did you ever drop or withdraw from a math course? ☐ No ☐ Yes; if yes, what course(s) and why did you drop it or withdraw? _____

14. As a student taking tests (not math), were you generally:

☐ Very relaxed ☐ Somewhat relaxed ☐ Somewhat anxious ☐ Very anxious

15. As a student taking math tests, were you generally:

☐ Very relaxed ☐ Somewhat relaxed ☐ Somewhat anxious ☐ Very anxious

16. How frequently did (do) you feel like giving up your study of math?

☐ Very rarely ☐ Somewhat rarely ☐ Somewhat often ☐ Very often

17. How frequently do (did) you feel like giving up the teaching of math?

☐ Very rarely ☐ Somewhat rarely ☐ Somewhat often ☐ Very often

18. Do you anticipate a change in career--either away from math or away from teaching math--in the future? ☐ No ☐ Yes

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THE TOY BOAT

A steamboat travels upstream for one mile at which time a toy boat is dropped into the water. The toy boat travels downstream with the current while the steamboat continues upstream for 20 minutes, turns around and arrives back at the starting point at the same time as the toy boat. Find the rate of the current.

At a gathering of mathematicians, someone delivered a paper on the meaning of Einstein's theories. After he had run on tediously for nearly an hour, someone interrupted to say: "I think you are greater than Einstein himself; twelve people understand Einstein--but nobody understands you." From: Edmund Fuller, (Ed.), 2500 Anecdotes for All Occasions. New York: Avenel Books, 1978, p. 198.

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ADDITIONAL READINGS

In 1979 an Ad Hoc committee was formed by the National Academy of Sciences to consider the current state and future requirements of the mathematics curriculum in colleges and universities. Special attention was given both to the mathematics major and to the need for all students to experience using and applying mathematical concepts. One of the conclusions the committee reached was that "the principle problem is one of attitudes": The committee's final report states "we have been made aware of the widespread existence of attitudes that must be substantially modified if students are to be encouraged to view positively the prospect of a career devoted to the use of mathematics in the solution of problems that arise in science, in industry, or in government service." (Ad Hoc Committee on Applied Mathematics Training, Assembly of Mathematical and Physical Sciences, National Research Council. The Role of Applications in the Undergraduate Mathematics Curriculum. Washington, D.C.: National Academy of Sciences, 1979, p.1.)

The concern with attitudes toward mathematics, although evident in the literature for a number of years, has only recently begun to gain the attention of a wide audience. The research relating to attitudes was reviewed in 1970 by Lewis R. Aiken, Jr. (Attitudes Toward Mathematics, Review of Educational Research, October 1970, 40 (4), 551-596.) Aiken organized his review around eight areas addressed by the literature: methods of measuring attitudes toward mathematics, grade distribution and stability of attitudes, relationship between attitudes and achievement in mathematics, relationship between attitudes and personality factors, teacher characteristics and behavior, instructional methods and curriculum, program efforts to develop positive attitudes, and lastly, suggestions for further study. Aiken's review of more than 70 publications is still the most comprehensive overview of the field available to date.

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He supplemented this with an update in 1976 covering the intervening five years. (Lewis R. Aiken, Jr. Update on Attitudes and Other Affective Variables in Learning Mathematics, Review of Educational Research, Spring 1976, 46 (2), 293-311.)

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One of the major research concerns has been to develop adequate methods to measure the level and intensity of math anxiety in school populations. Some of the earliest efforts in this regard were designed by Wilbur D. Dutton. Two of the more recently developed and validated measures are the Fennema-Sherman Mathematics Attitudes Scales and the Mathematics Anxiety Rating Scale (MARS). All of these instruments are self-administered questionnaires.

- Wilbur H. Dutton. Measuring Attitudes Toward Arithmetic, Elementary School Journal, September 1954, 55, 24-31.
- Wilbur H. Dutton. Attitudes of Junior High School Pupils Toward Arithmetic, The School Review, January 1956, 64, 18-22.
- Elizabeth Fennema and Julia A. Sherman. Fennema-Sherman Mathematics Attitudes Scale: Instruments Designed to Measure Attitudes Toward the Learning of Mathematics by Females and Males, Catalog of Selected Documents in Psychology, 1976, 6 (2).
- L.R. Brush. A Validation Study of the Mathematics Anxiety Ratings Scale (MARS), Journal of Educational Psychological Measurement, 1978, 38, 485-490.
- Frank C. Richardson and Richard M. Suinn. The Mathematics Anxiety Rating Scale: Psychometric Data, Journal of Counseling Psychology, 1973, 19 (6), 551-554.
- Earl L. McCallon and John D. Brown. A Semantic Differential Instrument for Measuring Attitude Toward Mathematics, Journal of Experimental Education, Summer 1971, 39 (4), 69-72.

* * * *

Another area of major interest in the literature is sex differences as they relate to mathematical achievement and attitudes. A 1974 study by Hilton and Berglund is indicative of the findings. Using data from a longitudinal study of students from grade 2 through college they found that there was no sex difference in achievement up through grade 5. By grade 7, however, males begin to emerge with the higher achievement scores than females. This finding is consistent with the hypothesis that differences that do appear may be attributable to sex stereotyping. As male experience the social expectation

to be interested in science and mathematics, they begin to do better than females who learn that it is not necessary or even appropriate for them to do well in these areas. Similar findings are reported in terms of attitudes, with females exhibiting poorer attitudes than males towards math, but only after ninth grade. One study (Norman) found that although both male and female attitudes toward math became more negative as they progressed through school, females' attitudes worsened at a greater rate than the corresponding males.

- Thomas L. Hilton and Gosta W. Berglund. Sex Differences in Mathematics Achievement: A Longitudinal Study, Journal of Educational Research, 1974, 67, 231-238.
- Ralph D. Norman. Sex Differences in Attitudes Toward Arithmetic-Mathematics from Early Elementary School to College Levels, Journal of Psychology, 1977, 97, 47-256.
- Gloria L. Carey. Sex Differences in Problem-Solving Performance as a Function of Attitude Differences, Journal of Abnormal and Social Psychology, 1958, 56, 256-260.
- Elizabeth Fennema and Julia A. Sherman. Sexual Stereotyping and Mathematics, The Arithmetic Teacher, May 1977, 369-372.
- Elizabeth Fennema. Sex Differences in Mathematics-Learning: Why?, Elementary School Journal, December 1974, (3), 183-190.
- John Keeves. Differences Between the Sexes in Mathematics and Science Courses, International Review of Education, 1973, 19 (1), 47-63.
- James W. Michaels. Closing the Sex Gap in Math Performance by Altering Reward and Task Structures, Educational Technology, June 1978, 22-25.

* * * *

Sells investigated the relationship between high school preparation and college career curriculum. She found that among the top 12.5 percent of high school graduates admitted to the University of California at Berkeley, 43 percent of the entering males and 92 percent of the entering women did not have sufficient training in mathematics to pursue most of the career majors offered at the university and were effectively left to choose from the humanities, music, social work, elementary education, or guidance and counseling majors.

- Lucy W. Sells (Ed.). "Critical Points for Affirmative Action", In: Towards Affirmative Action #3, New Directions for Institutional Research, San Francisco: Jossey-Bass, 1974.

- Lucy W. Sells. Mathematics--A Critical Filter, The Science Teacher, February 1978, 45 (2), 10-12.

* * * *

Many programs have been designed to assist the math anxious and math avoider. One of the first and best known of these effects is the math clinic at Wesleyan University. The program includes the use of diagnostic and therapeutic interviews with math counselors and math teachers, the use of Gestalt techniques to deal with students with severe emotional conflicts associated with mathematics, a tutorial program, and specially designed substantive math classes many of which are co-taught by a mathematician and counselor. The clinic issues periodic reports and papers on their work. (Write to: Math Clinic, Room 547, Science Tower, Wesleyan University, Middletown, Conn. 06457.) Evaluative studies of some programs designed to deal with math anxiety are listed below.

- J. Spanier. Mathematics Clinic: An Innovative Approach to Realism Within an Academic Environment, American Mathematics Monthly, 1976, 83, 771-775.

- F.C. Richardson and R. Suinn. A Comparison of Traditional Systematic Desensitization, Accelerated Massed Desensitization, and Anxiety Management Training in the Treatment of Math Anxiety, Behavior Therapy, 1973, 4, 212-218.

- Nasim Dil and Edward Earl Gotts. Improvement of Arithmetic Self-Concept Through Combined Reinforcement, Peer Interaction, and Sequential Curriculum, Journal of School Psychology, 1971, 9 (4), 462-472.

- R. Hunkler and W.G. Quast. Improving the Mathematics Attitudes of Prospective Elementary School Teachers, School Science and Mathematics, 1972, 72, 709-714.

* * * *

Recently the problems of math anxiety and math avoidance have received some recognition by the general public. Two new book-length treatments of the subject have been published. Both provide detailed discussions of the characteristics of the math anxious and the math avoider, some analysis of the characteristics of math that people find particularly troublesome, and

details on how to overcome one's sense of fear. Sheila Tobias, the author of Overcoming Math Anxiety (New York: W.W. Norton, 1978), is a counselor by training and has been closely associated with the work of the math clinic at Wesleyan University. Mind Over Math (N.Y.: The Dial Press, 1978; Paper edition, N.Y.: McGraw-Hill Book Co., 1979.) is by Stanley Kogelman and Joseph Warren. Kogelman has a Ph.D. in mathematics and an M.S.W. degree. He teaches mathematics and developed the Mind Over Math workshop program to work directly with people to overcome their fear of mathematics.

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MART: Mathematics Avoidance Reduction Techniques (III)

WORKSHOP SESSION #2

AGENDA

To prepare, review this section of the Leader's Guide
review Chapter 7 [5]
read Chapters 4 [3] and 5 [3/4]

Note: You will need a chalkboard or large newsprint
pad and marker

- A. Welcome
- B. More About Word Problems
 - translating the Direct Translation Method
 - a brief discussion of the pros and cons of this approach
- C. The Public's Image of Mathematics
 - how others view us
 - how we view ourselves
- D. Mathematics: What is it?
 - students' beliefs and perceptions:
 - the quest for the right answer
 - mathematicians' beliefs and perceptions:
 - beauty and elegance
 - cumulative nature
 - conceptual leaps

Materials for Distribution to Workshop Participants

- *Word problem, (Handout #2.1)
- *Word problem, (Handout #2.2)
- MART II: Chapters 4 and 5 [3 and 4]

*For use in Session #2

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A. WELCOME (15 minutes)

As participants arrive,

- ask them to sign in.
- discuss their reactions, if any, to the material distributed in Session #1.
 1. Deal first with reactions to the background Chapters (1 & 2; [1 & 2]). You don't, of course, have to defend this material. (Suggest, instead that they might make notes and share these with the MART Staff; we are interested in their reactions.)
 2. Should the discussion be about the actual extent or severity of math anxiety among students at your college, you might make the point that it may not matter: effective pedagogical approaches will work with all students and if with math anxious ones also, so much the better. Trying new ways also benefits instructors--it helps keep them from getting bored and frustrated.

B. MORE ABOUT WORD PROBLEMS (45 minutes)

Purpose: a continued discussion of the Direct Translation Method of solving word problems; to illustrate its effectiveness and to compare and contrast it with the approach typically taken by students.

- Start by talking about students' reactions to word problems.
 1. Generally, students don't like word problems.
 2. Word problems panic many students and are one of the things that cause math anxious people great distress.
 3. People attack word problems differently, but almost everyone reads them through--over and over again. Math anxious students believe that by doing so, the answer will "appear".
- Distribute Handout #2.1, and ask participants NOT TO READ THE PROBLEM THROUGH.

There are two reasons for telling students not to read word problems all the way through (or, if they are compelled to, to read a problem once very quickly:

- the first has to do with drama. No one ever told them this before, and by doing so, the instructor captures their attention.
- in addition, however, reading a whole problem is not only confusing and/or overwhelming, but it doesn't further the solving of it. The important thing is to get students to start working.

- Using the chalkboard, guide the group through the problem (see Chapter 7 [5]).

1. Read, "The second of three numbers" and stop. Since the problem is discussing three numbers we start a list of information. At this point, all that is known is:

First number =
Second number =
Third number =

2. Continue reading, "The second of three numbers is 1 less than the first." We study the sentence (without reading further) and since the second number is being discussed with reference to the first, we now fill in:

First number = n
Second number = $n - 1$ (since "one less" means the number less one.
If students have trouble writing this backwards, -1, (not 1-) use "25 dollars" as an example: \$25 (not 25\$).
Third number =

- There is still no information regarding the third number. However, doing the problem in this way is calming because there is only a small amount of information to focus on at any moment.

- It also leads us to anticipate what is coming next. In fact, we expect that we will now be told something about the third number.

3. We read the next phrase, "the third number is 5 less than--" Notice again that "5 less than" means something less 5 and we write

Third number = -5 (leaving a blank for whatever the third number is less than.)

4. Continuing, we read, "the third number is 5 less than twice the second." We look back at our list of information and see that the second number we are referring to is written as $n-1$. To translate "twice the second" we hold the second number together with a parenthesis and fill in:

Third number = $2(n - 1) - 5$

5. We go on to read that "the third number exceeds the first number by 12". To say that the third number exceeds the first is the same as saying that the third number is equal to the first plus something. From our list of information, we copy down the third number and the first number as written:

$2(n - 1) - 5 = n$ plus something

The "something" is given to be 12, so finally we have the equation:

$2(n - 1) - 5 = n + 12$

- Distribute Handout #2.2

1. If there is time and participants are interested, have the group work out the problem (or any other problem of their choosing) using the Direct Translation method.
2. Alternatively, you might suggest that they try this problem as "homework", or try another word problem using the same approach.

● Summarize the Direct Translation Method.

1. Doing math requires calm and confidence. This is especially true when it comes to solving word problems.
2. At each step, don't bite off more than you can chew.
 - reading the whole problem is overwhelming.
 - the prose is not meant to be understood the first time through, otherwise there would be no "problem"--just the mechanics of the solution.
 - deal with one word or one phrase at a time.
3. When in doubt, write it down.
 - using a separate piece of paper, write down all the information contained in each word or phrase that seems relevant or important.
 - most numbers will probably, but not invariably, be important.
 - certain key words tell the problem solver that something important follows: e.g., "from certain information..." and the "if... then" combination.
 - it is better to write down too much information than too little, remembering that you may not need it all.
4. Translate the important/relevant information into mathematical symbols and form. When possible, use abbreviations for objects, persons, or events.
5. Extract additional information from that which is given. Sometimes this means going outside the problem to your own repertoire of knowledge: for example, d = the number of dimes, but $10d$ = value of those dimes.
6. The desired equation or relationship should now be discernible.
 - if not, don't reread the problem first.
 - first go back and reexamine what you wrote.
 - each bit of information should be judged in terms of whether it contributes to the solution.
 - do not be afraid of discarding information that is not pertinent.

7. If its' a "tricky" not a "trick" problem:
 - the solution or the insight that unlocks it should be apparent.
 - if not, experiment with the numbers; use your intuition and creativity.
 - if you still have no success, go away from the problem for a while. After doing something else, start over again.
- Summarize participants' reactions, indicating that:
 1. When they are introducing word problems in a class, start by using examples the students come up with. As an illustration, ask "who went out last night?", and develop a problem using the experience.
 2. Don't try to make up funny problems for students. They won't think they are funny. If the instructor can do things in a funny way, that helps.
 3. The Direct Translation method can be applied to word problems at every level of difficulty.
 4. This method stops the problem solver from floundering aimlessly--and thereby decreases anxiety or tension.
 5. The Direct Translation approach also helps problem-solvers set up the problem in a form that allows them to solve it.

C. THE PUBLIC'S IMAGE (20 minutes)

Purpose: to examine ways in which mathematicians' perceptions of mathematics (and of mathematicians) differ from those of the general populace; to explore why these differing views present problems in the classroom; and to begin to discuss what instructors might do to bridge the gap.

- Begin by stating the problem:

"Part of the math anxiety problem stems from the way in which non-mathematicians tend to perceive both mathematicians and mathematics."
- Encourage discussion about how mathematicians are viewed (Chapter 4 [3]).

Encourage participants to talk about their experiences in social settings when people discover they are mathematicians. You might start with an experience of your own, for example:

"I'm sure you've all had the experience of meeting someone at a party, telling him/her you're a mathematician, and having them say 'You don't seem like a mathematician. Math! Yuck! It was my worst subject. I can't do it.' Then they stand there wondering what they could possibly say to a mathematician. They seem to feel we have few 'human' interests and can't wait to get home and sit down with a good math book".

• Summarize the major points.

1. People use the similar words to describe mathematics and mathematicians: logical, cold, aloof and abstract (or abstracted), rigid or inflexible...as well as humorless, unrelated to reality, and disinterested in people.
2. Such perceptions are not only commonplace among students, but underlie many of the behaviors and negative attitudes we encounter in the classroom.
3. Unfortunately, when people perceive mathematicians as cold, insensitive, and disinterested--as being more comfortable with numbers than with people--they are less apt to approach a math instructor with questions or problems.
4. When they perceive mathematics as an alien and inhuman subject, there is little likelihood that they will want to deal with it at all.
5. The public's perceptions have not just sprung into being. It is often fostered by a traumatic incident involving math, and is fed by misconceptions about what math is and how it is done, on the one hand and, on the other, by certain qualities of mathematics itself.
 - The inevitability of a right answer to a problem, for instance, is viewed by most mathematicians as a positive virtue of math, but by many students as a distinctly negative attribute.
 - Most mathematicians find math to be creative; most students never develop the practice and level of mastery that would enable them to either work on creative problems or to approach more traditional problems in more creative ways.
 - As another example, while mathematicians recognize that math is a "cumulative" discipline, that quality is not self-evident to students who more often view it as a random collection of discrete topics. (e.g., Students who haven't mastered fractions don't understand why they are having trouble with decimals.)
6. Students' perceptions and misconceptions need to be directly addressed in the classroom. This involves the instructor in discussions of such things as how to study math, the interrelationship of topics, and so on.

D. WHAT IS MATH? (40 minutes)

Purpose: to reinforce the differences between the views of mathematicians and those of the general public; and to indicate how and why some of our assumptions about math affect the way we teach. (Chapter 5 [3/4]).

● Have participants list what they like about math.

1. Usually, math instructors will mention, among other things, the beauty and elegance of mathematics, it's internal logic and consistency, and the symmetry. While the participants may not use these terms, much of what they say is easily recognizable by these words.
2. Some may describe the fact that they always felt comfortable--that is, they always knew where they stood--in math classes. Others may choose to employ phrases like, "math is objective". We feel this refers to the fact that math problems have an answer--unlike problems in the humanities where "opinion" is very often the solution.
 - many people react negatively to this characteristic which they experience as rigidity, inflexibility, uncreativity, mechanistic, and inhibiting.
 - if instructors take active cognizance of this, they will easily rediscover ways of illustrating that doing math can be creative. Acknowledging a student's unusual and correct approach is just one example.
3. Math also has other attributes that sets it apart from other disciplines:
 - math is a particularly cumulative subject; some people like the interrelationship of topics and the fact that concepts build on one another, while others both fail to see the connections or don't appreciate them. Instructors need to remember that most people don't know this about mathematics.
 - the other characteristic of mathematics, important for understanding the difficulties many people have with math (including those of the math anxious), has to do with conceptual leaps (see below and Chapter 8 [6]). While we can reconstruct the conceptual leaps in calculus, it is difficult for us to remember that they occur in arithmetic too.

● Summarize the discussion.

1. Point out that many of the qualities about math which mathematicians perceive and which contribute to their enjoyment of math are rarely visible to students, in large part because they are difficult to convey.
2. Propose that participants consider trying to share their enjoyment and excitement with their students.
3. The following are examples that can be used in elementary courses to convey the neat, logical, and uncomplicated way (the beauty and elegance) in which solutions or proofs are arrived at.

Numbers divisible by 3 or 9

A number is divisible by 3 or 9 if the sum of the digits is divisible by 3 or 9. (It may be better to do the proof before stating the result. Frequently, students have difficulty understanding the need for "proof" once the result is known and they are convinced that it works.)

- For students without a background in algebra, it is best to do the proof by example:

- Consider the number 3,516.

- Rewrite the number as:

$$3,516 = 3 \times 1,000 + 5 \times 100 + 1 \times 10 + 6.$$

- We then do a mathematician's "trick" and write the numbers 10, 100, etc. in another form. (Here it is useful to emphasize that we can "play" with numbers and through experience we learn many useful ways to manipulate numbers so that we learn more about them.)

$$\begin{aligned} 3,516 &= 3x(999 + 1) + 5x(99 + 1) + 1x(9 + 1) + 6 \\ &= 3 \times 999 + 3 \times 1 + 5 \times 99 + 5 \times 1 + 1 \times 9 + 1 + 6 \\ &= 3 \times 999 + 5 \times 99 + 1 \times 9 + 3 + 5 + 1 + 6 \end{aligned}$$

- Now we make the observation that the number can be divided into two groups, one that is divisible by 3 and by 9 and a second group that is really the sum of the original digits. Underscore that an important part of doing math is sitting back and making observations and finding patterns.

Adding numbers 1 through n

- ask students to see if they can "discover" a formula for adding up the numbers 1-10. Write down their suggestions.

- then explain that working on this problem for a long time might lead to the following observation:

$$\text{Sum} = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$

also

$$\text{Sum} = 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$$

- adding vertically we find that:

2 sums = 11 + 11 + 11 + ... (a total of ten 11's), so the actual sum is 1/2 (11 x 10).

- By an ingenious but simple maneuver we have discovered a neat way of adding any series of consecutive numbers.

- If there is time, introduce the notion of conceptual "leaps of faith".
(If there is insufficient time, it can be covered in Session #5).
 1. Learning mathematics requires leaps of faith or the "suspension of disbelief".
 2. Some are easily recognizable; most of us vividly remember the difficulty we had learning the limits concept. The traditional example used to illustrate this concept is to consider how long it would take a turtle to cover a mile if each day he walked exactly half the remaining distance.
 - Since the turtle walks "only" half the remaining distance each day, another half always remains and therefore he never reaches his goal. The problem is that when we try to imagine the turtle we know that when the distances start to get smaller and smaller it becomes physically impossible for him to keep moving without appearing to reach the wall.
 - The difficulty in the above concept is that we have to extrapolate from physical reality and imagine an "infinitesimally thin" turtle. So, in essence, understanding the concept of infinity requires us to understand the concept of infinitesimally small before we start. Once we have understood the concept, it is difficult to understand why we had such trouble "getting it" in the first place.
 3. There are other less exalted occasions when a "leap of faith" is required of the learner:
 - use of the symbol "-" to stand for negative numbers and the concept of negative numbers
 - fractions such as $5/13$ which we can't readily visualize
 - the use of letters to stand for numbers
 - the use of numerals 1, 2, 3... to represent abstractly the idea of two things or three things, but without those "things" being present or visible.
- Emphasize that it is not always apparent that such a leap is called for.
 1. There are certain tell-tale signs that signal it: for example, students appear confused and repeatedly ask the same question.
 2. There are many things teachers can do to assist students to make these leaps:
 - Start with a sequence of illustrative examples before discussing the concept.
 - Point out to the class that a new concept is being discussed and that it is going to require a conceptual leap.

- Provide examples to illustrate what a conceptual leap is (using "3" to stand for three things but without writing those things).
 - Clearly state the concept they are being required to learn (e.g., "We are now, in effect, grappling with the concept of infinity.")
 - Allow time for class discussion and objections. Frequently, a member of the class can better understand another student's difficulties and can translate a question in a way that helps you understand what the question is so that you can then answer it.
 - Underscore that learning a new concept will take time and practice.
 - Try not to be impatient and show constant faith in everyone's ability to ultimately master the new idea.
 - Provide a variety of examples and hope that one of them will click for each student.
- Distribute copies of Chapters 4 and 5 [3 and 4]).
 1. Explain that these chapters cover much of the material that was discussed in today's Session #2).
 2. Briefly mention that next time they will be talking more about math anxious students and how to relate to them in the classroom.
 3. Session #3 will also cover fractions. In anticipation, ask for a volunteer who would like to show the group how he/she teaches fractions. Ask him/her to prepare a 15-20 minute presentation.

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MART: Mathematics Avoidance Reduction Techniques (III)

Problem 1

The second of three numbers is 1 less than the first. The third number is 5 less than twice the second. If the third number exceeds the first number by 12, find the three numbers.

"When colts can't work math problems, do their teachers urge, 'Just use your people sense'." Anonymous

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MART: Mathematics Avoidance Reduction Techniques (III)

Problem 2

Mildred bought 2¢ stamps, 4¢ stamps, and 5¢ stamps for \$4.35. The number of 2¢ stamps exceeded the number of 5¢ stamps by 50. The number of 4¢ stamps was 10 less than twice the number of 5¢ stamps. How many of each kind did she buy?

"The different branches of Arithmetic--Ambition, Distraction, Uglification, and Derision," The Mock Turtle in Alice in Wonderland by Lewis Carroll.

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MART: Mathematics Avoidance Reduction Techniques (III)

WORKSHOP SESSION #3

AGENDA

To prepare, review this section of the Leader's Guide
read Chapters 6, and 9 [4 and 7]
review Chapters 1,3 [1,2]

Note: You will need a chalkboard or large newsprint
pad and marker

- A. Welcome
- B. Characteristics of Math Anxious Students
 - visible symptoms
 - etiology and effects on learning
- C. Fractions
 - a significant problem area
 - comparison of different teaching strategies
- D. Do's and Don'ts in the Classroom
 - general techniques and tactics
 - specific suggestions
 - some topics for classroom discussion

Materials for Distribution to Workshop Participants

- * Handout # 3.1, "Hard Fractions"
- Chapter 6 [4]
- Chapter 9 [7]

* For use in Session #3

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A. WELCOME (10 minutes)

As participants arrive,

- ask them to sign in.
 - discuss their reactions, if any, to the material distributed in Session #2.
1. Spend a few minutes discussing their reactions as a group.
(Try to separate responses about writing style or workshop format from responses that deal with MART's ideas and concepts.)

B. CHARACTERISTICS OF MATH ANXIOUS STUDENTS (30 minutes)

Purpose: to define math anxiety more precisely by focusing on how it is manifested in a wide range of student behaviors and attitudes.

- What math anxious students "look" like (see Chapter 1 [1]).
1. Mention that the group might spend a few minutes tying together the discussion of math anxiety that took place during Sessions #1 and #2, and the materials that were distributed.
 2. The purpose of this summary is to clarify the characteristics of math anxious students in order to begin to deal more efficaciously with their needs.
 3. Have participants begin to enumerate the physical (i.e., external) symptoms that often accompany math anxiety that suggest its presence.
 - Remind them that in many of its external manifestations (i.e., those qualities that others can observe), math anxiety "looks like" general anxiety.
 - Stress that participants recall their own physical reactions in mathematical situations that they experienced as anxiety provoking.
 - Symptoms range widely, reflecting both the acuteness of the anxiety as well as the personality of the student. They include: sweaty palms, palpitations, stomach aches or headaches; giving up at the first sign of difficulty, withdrawal, absenteeism or lateness; frequent nervous laughter or restlessness; lack of eye contact with the instructor, apparent inability to pay attention, disrupting other students, or directly antagonistic behavior.

- The easiest way to diagnose if students are math anxious is by asking them. Math anxiety is at a level of awareness--the victim knows it--and is usually willing (even eager) to talk about it.
4. In general, obvious and extreme symptoms are exhibited by students with acute (most debilitating) math anxiety. As one moves to the opposite end of the math anxiety spectrum, however, math anxiety's effect on individual performance is still clear, but it's expression in public behavior is less evident.
 5. Math anxiety is no respecter of persons.
 - The math anxious person can be anywhere from 13 to 70 years old. There is no typical age, but people recall it beginning early in their lives, usually associated with a specific math topic or teacher.
 - While there is some uncertainty whether math anxiety is more prevalent among women, women tend to avoid math more than men.
 - Adult victims are frequently of above average intelligence. Many have had at least some college education or have attained advanced degrees in non-technical (i.e., non-math based) fields such as law, medicine, English, anthropology...
 - Math anxiety is also found in math majors and mathematicians (remember the Toy Boat Problem), but is also prevalent among people poorly prepared in math.
 6. Other than physical symptoms, and more devastating in its effects on learning, are the emotional and attitudinal concomitants of math anxiety:
 - math anxious believe they are simply dumb when it comes to math and, by extension, believe that they will never be able to do it.
 - math anxious people attribute their stupidity about math to biology, including gender ("I don't have a 'math mind', nor did my mother"), and not to attitudinal factors. When pressed, however, they will use any peg to hang their math problem on.
 - most math anxious people can recall traumatic episodes with teachers in elementary or junior high school--being called to the blackboard and humiliated in front of classmates or being subjected to ridicule or sarcasm for doing math "wrong". Timed tests, speed drills, and flash cards (rote memorization) are also singled-out for their long-lasting, far-reaching negative effects.
 - fractions represent a particular point of difficulty for most all math anxious people. So do word problems. The math they need is usually everyday math.

- when questioned, math anxious people are very articulate about their problems; nevertheless, they are fearful that others will find out how stupid they are. As a result, they do not ask just those questions that would likely help them the most.
- feelings of incompetency regarding math may extend to other things like mechanical skills and reading charts and diagrams.

● What math anxious students "act" like.

1. Math anxious people invariably seek ways to avoid math. They postpone meeting math requirements, avoid math courses that would permit them to enter a wider range of careers, and try to get out of every day situations involving math.
2. Math anxious people believe that math is done by magic. They believe many math myths and engage in certain destructive games that they play on themselves.
3. They perceive math as rigid and authoritarian. They feel that math offers no room for creativity.
4. Their tension causes them to make careless mistakes and to write down obviously incorrect answers.
5. They often exhibit an apparent unwillingness to think (they experience it as being "unable to think"), while there is no impairment of their reasoning, abstracting, or logic in other areas.
6. They demand rules and set procedures.
7. When questioning the instructor, they jump from question to question.
8. They exhibit a general impatience and desire to skip over things.
9. They tend to be unable to correctly copy their work from step to step. Transposition errors are very frequent.
10. Many math anxious people have a strong desire to overcome their handicap, but this is often coupled with an equally strong feeling of hopelessness about succeeding.

● The core of math anxiety is a lack of confidence in one's ability to do math.

Therefore, the primary ameliorative thrust of instruction must attend to this underlying condition in order to make further learning of math possible.

C. FRACTIONS (45 minutes)

Purpose: the purpose of this component of the program is to explore why and how fractions cause many people difficulty and to examine approaches to teaching what will help communicate the concepts more effectively.

- Fractions are a widespread problem (see Chapter 9 [7]).

Introduce the topic of fractions by commenting that they cause very many people difficulty, even people who are not particularly math anxious. You might want to say something like:

"If we were to choose one area of mathematics that presents the most difficulty for the most people it would be fractions. Most adults are uncertain about the meaning of fractions and about the arithmetic operations used when working with them."

- A persistent question indicating lack of understanding is "Why does multiplication of fractions lead to smaller numbers when multiplication means 'get bigger'?"
- In addition, most people believe there are two kinds of fractions: easy ones ($1/2$ or $1/4$) and hard ones ($3/7$ or $\frac{11}{13}$).
- Distribute "Hard Fractions" (Handout #3.1) and have participants solve it.

- To address students' perceptions and clear up their misconceptions requires attention to very specific techniques.

Discuss each of the following points and encourage participants to describe other methods they use to accomplish the same purpose:

1. Teaching fractions (whether for the first time or anew to students) requires a return to concrete visualization.
2. Some "visualizations" are better than others: pies (rather than rectangles) have distinct advantages, even if college students feel they are babyish". (Rectangles are not an effective mental image because when cut into pieces, the pieces still look like rectangles.)
3. To avoid the feeling of intimidation students often experience when they confront a quantity like $5/7$, they should learn how to read a fraction in a way that conveys it's meaning. For example, they should learn to read $5/7$ as "5 of those things called sevenths" or "a whole has been divided into 7 equal parts and I have 5 of them."
4. A conceptual "leap of faith" is required for the student to see that "in principle" $1/2$ is the same as $1/17$. Practice is required before conceptual leaps are possible.
5. The language used to explain operations is always important. It is best to avoid words like "numerator" and "denominator". Save them for the end of the discussion.
6. Before adding fractions, have students compare them. Ask which is bigger, $1/7$ or $1/8$? Which is bigger, $8/9$ or $9/10$ (teach them to look at what's left over ?

7. Addition of fractions appears to be "more natural" if one thinks of the fractions initially in words. For example: 3 of those things called eighths plus 4 of those things called eighths equals 7 of those things called eighths.
8. It is often easier for students to add fractions with unlike denominators vertically rather than horizontally. When teaching addition of fractions examples should initially be done both ways.
9. When converting fractions, as for example, $3/4 = ?/12$, have students practice saying "I've tripled the number of pieces the whole was cut into, so I must triple the number of pieces I have--9/12."
10. Multiplication of fractions is difficult because when dealing with fractions the word (times) and symbol (X) convey a meaning different from what students learned when working with whole numbers. Care must be taken to recognize and explain that multiplying fractions is different than multiplying whole numbers. This confusion can be surmounted by describing the multiplication of fractions as "of-ing" (i.e., you are taking a fraction of a fraction).
11. Division of fractions, while operationally easy, is intuitively difficult to understand and to explain. (You might want to consider talking about division as "unmultiplying"). One has only three choices in teaching division of fractions: you can tell students the rule, you can demonstrate how to do it, or you can "twist around" their thinking. It is helpful to emphasize that, for example, $1/2 \div 1/4$ asks the question "how many fourths are there in one half?" The answer follows when they reason that "there are 4 fourths in a whole, so there $1/2$ of 4, or 2--2 fourths in a half."

• Alternative ways of teaching fractions.

1. If there has been insufficient group involvement, you might ask for a volunteer to demonstrate his/her methods. (Or, if you already asked someone to prepare, be sure you follow up on it.)
2. After the presentation, you might want to indicate that while the approach was good, it too created problems.
3. Ask the group to compare and contrast the approaches.

• Conclude that there is no perfect approach for teaching fractions or any other math concept or operation.

1. The desirable approach is one that causes the least confusion; is mathematically sound but no more complex than absolutely necessary (e.g., it is not necessary to introduce prime factors, LCD's, or the commutative law into a discussion of fractions); focuses on one operation at a time, in the order they are needed to be able to proceed, and in a way that sets each different operation apart; and that doesn't feed into students' misconceptions and that is non-threatening to them.

2. When teaching something like fractions, which all students have been exposed to countless times (and didn't learn), the instructor must find new ways to capture and hold their attention: otherwise, they will latch on to a word or phrase they think they know and will tune you out. So, for example,

- don't use numerator or denominator at first.
- talk about "points of common happiness" rather than lowest common denominator--learning to find the point of common happiness by multiplying two denominators together will be helpful in algebra.
- in actuality, of-ing describes the concept (and size of the resultant answer) more accurately than multiplying.
- similarly, unmultiplying is also more descriptive, novel, and perhaps even inherently funny.

D. DO'S AND DONT'S (30 minutes)

Purpose: to present simple but effective things that can be tried in the classroom to reduce the level of math anxiety by dissolving the image of math as magical and mathematicians as alien.

- Mathematicians are perceived as not quite human (see Chapter 6[4]).
 1. Personalize yourself.
 - Preferably at the first class meeting, talk about your background.
 - Stress the times when you had difficulty with math and thought that you might not be able to master the content.
 - Students will be better able to identify with you and will begin to understand that math actually does become progressively more difficult, even for mathematicians.
 2. Always write things down, covering all steps. This helps discourage memorization of rules and procedures.
 3. De-emphasize mental arithmetic and calculations--yours and theirs--to show that math is not best "done in one's head".
 4. Say the steps aloud and tell students to do so themselves when they are studying or solving a problem.
 5. When writing on the board, do not erase inadvertant errors immediately. Stop and explain what happened, but make certain students don't copy it.
- Math anxious students think they're alone.

1. Although the mere mention of "math" often provokes giggles of discomfort out of class, in the classroom most, math anxious students are not aware of how widespread their reactions and experiences are.
 - get students to talk about their past experiences with math.
 - encourage them to feel free to discuss their feelings during the course.
 - try having students keep a "math autobiography" or diary in which they detail their feelings on a daily basis.
 2. Math is usually not taught or done in groups. Therefore, individuals have very little opportunity to discuss how they arrived at solutions to problems.
 - try any technique that helps make the process of math more apparent.
 - let students do some work in small groups.
 - always invite students to describe alternate solutions or approaches that work. (This also helps them see that math can be creative.)
- Do not take things for granted.
1. The very things that instructors tend to like about math most students tend not to like.
 - many mathematicians like the fact that math problems have the security and objectivity of only one correct answer; most other people find this constraining, inflexible, and sometimes unfair.
 - to reduce their perception that math is rigid and authoritarian, encourage students to question and challenge as they do in other subject areas.
 - despite the awkwardness of an approach, avoid telling students "This is a better way to do it: it is more helpful to say "This is just one other way to do it."
 - directly address the appreciation-gap that exists between you and most of your students. If you find something neat, elegant, or exciting, say so--but don't expect them to necessarily buy it.
 - provide students with an overview of mathematical subjects and how they fit together. This will help them appreciate its cumulative nature and will also demonstrate that even very difficult math builds on basic manipulations.

2. Mathematics is a new language in its vocabulary and syntax.

- students need to understand that although math is very precise, the meaning of the words it uses differs from the everyday meanings of the same words.
- do not assume that the notation you are using, or that of the text, is familiar to all students. Spend a few moments on rotation.
- some problems, especially word problems, require knowledge of things not in the statement of the problem per se. Students need practice is how to bring information of this kind to a solution.

• Math anxious students feel incompetent in math.

1. Show the same interest in and liking for students who do poorly as for those who do well.
2. Answer all questions, including those you just finished answering and those that students "should know" the answers to.
3. Avoid singling anyone out.
 - always ask for volunteers
 - try not to call on students who are uncomfortable working at the blackboard.
4. Speak privately to students who seem to be having particular difficulty.
5. Try not to foster feelings of competition among members of the class. Mathematics has a built-in competition between the person solving the problem and the fact that there is a solution.
6. Encourage students to keep at a problem.
 - point out to them what is constructive about their attempted solutions.
 - discuss the difference between taking a break and giving up.
7. Discuss the self-fulfilling implications of a person believing or having been told she/he can't do math.
 - show them how much math they actually do as part of daily life.

• Math IS a difficult subject.

1. All people find math difficult--but at different levels.
 - avoid telling student's something is "obvious".
 - avoid telling students something is "easy"--someone in the class won't find it so.

- similarly, avoid telling them is "hard"--expecting it to be hard will discourage math anxious students from trying.
2. Although practice is important in learning math, don't tell students they will do better if they work harder. They will do better if they work better.
 3. Present illustrative sketches and diagrams before discussing procedures.
 4. In introducing a new topic, start out with examples based on intuition. Then begin to develop procedures.
 5. Do not easily give in to the temptation to show students several different versions of an approach or solution. They will try to reconcile them and become more confused. At the same time, however, keep in mind that "another way" might work for some students.
 6. When appropriate, as for example in showing the derivation of a formula, remember to tell students that some one or more mathematicians spent a great many years "getting it."

E. CLOSING (5 minutes)

- Any topic not covered in sufficient depth can be returned to in subsequent sessions or in additional sessions.
- Distribute copies of:

Chapters 9 [7] and 6 [4]
- Remind participants of the date, time, and place of Session #4.
Describe the agenda for Session #4 (Math Myths, Signed Numbers, and Topics for Classroom Discussion).

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Project MART

HARD FRACTIONS

Find 7 distinct, positive integers -- a, b, c, d, e, f, g -- such that

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{1}{e} + \frac{1}{f} + \frac{1}{g} = 1$$

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MART: Mathematics Avoidance Reduction Techniques (III)

WORKSHOP SESSION #4
AGENDA

To prepare, review this section of the Leader's Guide
read Chapters 12 [10] and 11 [9]
review Chapters 5, 6, and 7 [3, 4, and 5].

Note: You will need a chalkboard or large newsprint
pad and marker.

- A. Welcome
- B. Math Myths and Their Consequences
 - myths people believe in
 - the implications of misconceptions about how math is done
- C. Signed Numbers
 - a significant problem area
 - comparison of different teaching strategies
- D. Topics for Classroom Discussion
 - math is a cumulative subject
 - math is an acquirable skill (study habits)

Materials for Distribution to Workshop Participants

- *Math Myths (Handout #4.1)
- Chapters 11 [9] and 12 [10]

*For use in Session #4

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A. WELCOME (10 minutes)

As participants arrive,

- ask them to sign in.
- discuss their reactions, if any, to the material distributed in Session #3.
- discuss with the group whether they are ready to continue or would prefer to spend time now catching up.

B. MATH MYTHS AND THEIR CONSEQUENCES (30 minutes)

Purpose: to examine some of the beliefs people bring with them to the study of math and to explore some of their consequences in terms of the maleffects on learning; the objective is not to convince people that the statements are false.

- Remind participants of the repeated references to math myths.
 1. Math myths are beliefs about the processes of math--beliefs about how math is or should be done.
 2. The myths are shared by a great many people. Math anxious people particularly believe in these (and other) myths--they do not have a good grasp about how math can be done.
 3. There is some evidence that belief in myths is associated with poor performance in math and a tendency to avoid it.
- Start the discussion by asking participants how many would agree that it is bad to count on your fingers?
 1. Ask how many remember being forbidden to count on their fingers?
 2. How many do it secretly?
- Distribute the list of Math Myths (Handout #4.1), and comment.
 1. Belief in myths stultifies creativity and inhibits experimentation.
 2. Myths concern the processes of mathematics as well as some supposed attributes of people who are good in math.
- It's bad to count on your fingers (myth #1).
 1. If one counts on her/his fingers, and has come to believe it's bad:
 - you feel embarrassed and guilty.
 - you come to think that you're not good in math and that other people are better.
 - you learn that speed is what is important in math and the way to do math is by memorization and rote.

2. Point out that there are certain situations in which everyone uses fingers (in determining the number of months between October and April, for example).
 3. People who count on their fingers and do the computations correctly are demonstrating an understanding of the concept and of the manipulation. Finger counting works and is productive. With help, the finger counter will make the gradual transition away from using fingers (but will return to it under pressure).
- Go over the myths one at a time, primarily to examine the implications of believing in them.
 - Men are better in math than women (Myth #2).
 1. If you are a woman and believe this:
 - you accept you can't do math as part of the concept of femininity.
 - you stop trying and avoid math, thereby severely limiting your career options.
 - you give up control of math-based activities (such as finances) and become dependent on others.
 2. If you are a man:
 - you put women down and indicate you don't really expect much of them when it comes to math or math-related activities.
 - Math requires logic, not intuition (#3).
 1. If you believe this myth:
 - you don't allow yourself to use intuition which is one of the strongest tools for getting started on problems.
 - you distrust solutions that are easy and that weren't forthcoming logically.
 - you expect logical, automatic solutions and when they don't come, you conclude you don't have the ability to do math.
 2. You set up pressure on yourself to do math in a certain way rather than trying to understand the problem and allowing it to dictate the approach.
 - In doing math you must always know how you got the answer (#4).
 1. If you get an answer, but can't explain it:
 - you doubt yourself and believe the answer must be wrong.
 2. You don't allow yourself the time to sit down and examine how you figured a problem out.

● Math is not creative (#5).

1. If you believe this myth, and see yourself as a creative person, you will not want to be involved with a non-creative subject like math.
2. You will distrust your intuition and try to do math mechanistically.
3. You will try to memorize solutions and approaches.

● There is a best way to do a math problem (#6).

1. If you believe there is a best way:
 - you won't be able to develop your own method of solution.
 - your method will always seem less than ideal and you'll think that everyone else has a better way.
2. You will think your method is wrong because it was not good enough.
3. You will hesitate to start a problem because you fear you will do it the wrong way.
4. You will try to remember how to do a problem rather than just doing it any way that you can.

● It's always important to get the answer exactly right (#7).

1. You will be unable to allow yourself the flexibility and insight that comes from approximation.
2. You will focus on the negative (the wrong part) rather than on the positive (the right part) of the solution.
3. When making a mistake you will feel totally defeated.

● Math requires a good memory (#8).

1. If you believe this myth:
 - and keep trying to memorize things, you will become increasingly confused.
2. You will perform poorly since memory is one of the first things to go under tension.
3. You will perceive math as a bunch of isolated facts and it will always seem like a foreign language that you haven't learned.

● Math is done by working intensely until the problem is solved (#9).

1. Believing this stops you from obtaining the benefits (particularly the insights) that come from taking breaks.

- you never learn the difference between "I can't do it" and "I can't do it now."
- 2. When you work too long (at one time) at finding a solution,
 - you get frustrated
 - you feel like you're wasting your time
 - you give up.
- Some people have a "math mind" and some don't (#10)
 1. Believing that a "math mind" is biological rather than learned causes you to get discouraged when math doesn't come easily.
 - and leads you to give up without putting in the time, effort, and concentration it takes to learn math.
 2. Each failure becomes a confirmation that you don't have the "right kind of mind" and
 - you become too quick to let others do it for you or help you rather than relying on your own ability.
- There is a magic key to doing math (#11).
 1. If you believe this myth
 - you will always be searching for the magic "formula."
 - you will not use your intuitive sense.
 2. You will keep trying to remember what to do rather than trying to figure the problem out.
- Mathematicians do problems quickly in their heads (#12).
 1. Subscribing to this notion:
 - leads you to try to do math problems faster than you are ready to do math.
 - decreases the importance of the fact that speed is gained through practice.
 2. Leads you to feel defeated before you even start.
- Summarize:
 1. Ask the group to come up with a list of additional myths.
 2. Point out that these twelve and other math myths are part of our folklore.
 3. Many people come to the study of mathematics with these pre-conceived ideas about what mathematics is all about and what the "right" way to do it is.

4. With math anxious people especially, great care must be taken not only to not reinforce the myths in class (e.g., by the instructor doing computations in her/his head), but to try to undo them both by example and by discussion.

C. SIGNED NUMBERS (45 minutes)

Purpose: the purpose of including signed numbers is to explore why and how they cause many people difficulty and to examine approaches to teaching them that will help communicate the concepts more effectively.

- Turn the group to a discussion of signed numbers.

You might say, "Signed numbers are basic to all the work in algebra. They present students (and teachers) with almost insurmountable difficulties. Indeed, one of the clearest pieces of evidence that there is such a thing as math anxiety is the fact that students who have no difficulty memorizing other things are simply unable to remember that when you multiply signed numbers, "like signs give + and unlike signs give - answers".

- Examine the difficulties inherent in presenting signed numbers.

1. The first difficulty is that the symbol "-" is being simultaneously used to mean several different things (e.g., minus, take away, opposite, to the left, below, and negative).
 - This is a major source of confusion for the student who is trying to memorize what to do.
2. The other major difficulty is that multiplication of two negative numbers does not have any obvious physical meaning.
 - It is a mathematical construct whose definition was motivated by a need for consistency with and generalization from other concepts.

- MART's suggestions for teaching signed numbers:

1. represent a desire to successfully communicate information to students.
2. take into account students' attitudes, interests, and concerns.
3. is not a perfect method, but is one that minimizes student difficulty.

- It is helpful to make the first approach to signed numbers intuitive.

1. First present negative numbers in terms of temperature or debits.
 - show how in these examples a negative can be interpreted as meaning opposite and that it also connotes direction.
2. Emphasize that the symbol "-" has many meanings or interpretations depending on the context in which it is used. Illustrate that the opposite of the opposite (-) (-), is +.

3. Encourage discussion of students' objections.
- Start with addition.
 1. Discuss addition as a reconciliation of accounts (debits/credits).
 2. This illustrates that the symbol "+" has several interpretations and that the addition of signed numbers is not the same as ordinary addition.
 - Skip subtraction and go onto multiplication.
 1. Students usually have no difficulty multiplying a positive by a positive number.
 2. Continue with multiplication of unlike signs.
 - In terms of debits, $2(-3)$ can be interpreted as two debts of three, resulting in a total debt of 6, which is written as -6.
 3. Emphasize that $2(-3) = (+2)(-3) = (-3)(+2) = -6$.
 4. Give students ample opportunity to practice.
 5. Multiplication of a negative by a negative causes the most confusion.
 - Firmly establish $(+)(+)$, $(-)(+)$, and $(+)(-)$.
 - Students will accept that a "+" has been assigned to $(-)(-)$ because of consistency, simplicity, and utility.
 - Division should follow fairly easily once multiplication has been mastered.
 - Subtraction should be saved for last.
 1. First present subtraction operationally, following the rules for multiplication.
 - For example, $+3 - (-2) = +3 + 2 = 5$.
 2. The interpretation of subtraction has a different meaning when subtracting signed numbers.
 - In dealing with whole numbers, subtraction may be thought of as "take away".
 - With signed numbers, subtraction has the special meaning of directed distance: i.e., how far is it from one number to another number? The example, $+3 - (-2)$ means, "how far is it from -2 to +3?"

As you did with word problems and fractions,

- Have participants discuss their methods.
- Compare and contrast methods in terms of:

1. Their simplicity; to be mathematically correct does not necessarily mean to be complex. The most simple approach is the best one.
2. Their effects on math anxious students. Here it is particularly important to ask if confidence is being built up.

D. TOPICS FOR CLASSROOM DISCUSSION (30 minutes)

Purposes: to suggest topics instructors can discuss in class to reduce anxiety about and dislike for mathematics by addressing students misconceptions about mathematics; to establish a classroom environment that supports the further learning of mathematics. (See Chapters 5, 6, and 7; [3,4,65]).

Introduce this program component by repeating that,

- Most people believe in the myth of the mathematical mind.
 1. Among other things, this belief leads them to expect to do math effortlessly, quickly, and without pain--without working at it.
 2. Unless the misconception is addressed, people who find they can't do math this way conclude they "can't do it" or "don't have it".
 3. People who do well in math work hard and the effort they put in is appropriate, directed effort.
- Math is a cumulative subject (see Chapter 7 [5]).
 1. Spend some time discussing why mastery of a particular topic is important.
 - All (almost) mathematics is interrelated; one topic, concept, or manipulation is the foundation for others.
 - Failure to master fractions continues to haunt the learner even in calculus.
 2. Show them by example.
 - Algebra uses basic arithmetic manipulations.
 - For calculus, one must know some geometry and trigonometry.
 3. Reinforce new and strange symbols by having students make a list of all the symbols and words that they know that mean "add" or "multiply".
 4. Explain that in order to learn something new, students may have to go back and relearn some other more basic concept.

- indicate clearly how much help you are able to give and how, when, and where students can avail themselves of it (e.g., after class, in your office at prescribed times, etc.).
 - specify other assistance that is available to them (e.g., the math lab, tutors, etc.).
 - help them break down a concept or operation into its smaller, more manageable component parts.
 - carefully diagnose and prescribe exactly what it is you want them to brush up on. (Telling them to study algebra is overwhelming and not helpful; be specific and enumerate the concept/operations they need practice in.)
- Math is an acquirable skill (see Chapter 12 [10]).
 1. Compare the learning of math to other skills like learning to play the piano or learning to play tennis.
 - math requires practice.
 - math requires active involvement.
 2. Present them with ample opportunity to practice, and stress the importance of practice in gaining speed and accuracy, in increasing dexterity and comfort with numbers, and in mastering specific problems.
 - distribute worksheets of problems that focus on a particular operation or idea.
 - discuss the importance of doing homework. Have homework count toward their grade to reinforce the need for practice.
 3. Discuss how to keep working on a problem.
 - point out the difference between taking a break and giving up.
 - talk about the distribution of practice and of working at math to master it (like tennis, a little at a time but done consistently).
 - show them how to set achievable goals for themselves and how to reward themselves for accomplishing them.
 4. Use personal anecdotes liberally. Personal stories not only humanize the instructor and give students something with which to identify, but they illustrate most dramatically how success in mathematics is achieved. Therefore, be generous in sharing your own and others' successful study habits.
 5. "Warming-up" is an important study skill and test-taking skill.
 - starting with "easy" or familiar problems helps focus attention.

- easy problems (those that each of us individually feel are easy) guarantees success and therefore makes us feel confident.

- easy problems also decrease anxiety.

6. It is also necessary to be aware, and to help students become aware, of the fact that the time they spend on doing math reflects their own priorities.

- students can choose not to spend a needed or desirable amount of time at practicing math; they have to weigh this decision in terms of their other needs and obligations.

- however, the consequences of choosing not to spend sufficient time at math will be reflected in how much they master and, in turn, by their test scores and grades.

- Other important topics, test taking and reading a math textbook, will be discussed in subsequent sessions.

E. CLOSURE (5 minutes)

- Distribute copies of Chapters 11 [9] and 12 [10].
- Mention the date, time, and place of Session #5.

Center for Advanced Study in Education
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MART: Mathematics Avoidance Reduction Techniques (III)

MATH MYTHS*

Men are better in math than women.

Math requires logic, not intuition.

You must always know how you got the answer.

Math is not creative.

There is a best way to do a math problem.

It's always important to get the answer exactly right.

It's bad to count on your fingers.

Mathematicians do problems quickly, in their heads.

Math requires a good memory.

Math is done by working intensely until the problem is solved.

Some people have a "math mind" and some don't.

There is a magic key to doing math.

*Mind Over Math (used with permission of the author.)

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MART: Mathematics Avoidance Reduction Techniques (III)

WORKSHOP SESSION #5

AGENDA

To prepare, review this section of the Leader's Guide
review Chapter 6 [4]
read Chapter 8 [6]

Note: You will need a chalkboard or large newsprint
pad and marker

- A. Welcome
- B. General Teaching Practices
 - expectations and actuality
 - the global classroom environment
 - the first day of class
- C. Calculus
 - the concept of limits
 - leaps of faith

Materials for Distribution to Workshop Participants

- Chapter 8 [6]

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A. WELCOME (10 minutes)

As participants arrive,

- ask them to sign in.
- discuss their reactions to the content of Session #4 and to the materials distributed at the end.
- ascertain whether they are ready to continue, and if so, start with:

B. GENERAL TEACHING PRACTICES (60 minutes)

Purpose: to contrast instructors' expectations with the reality of the community college and to suggest ways in which their interest in their students can be the basis for teaching mathematics; and to describe specific actions instructors can initiate at the first class meeting that improve the environment so that more teaching/learning is possible for the majority of students who don't like math. (The entire point of this segment is to get a dialogue going so that participants make apparent to themselves and to colleagues what effective things they do every-day.)

- Most math instructors did not envision a career at a community college. See Chapter 4 [3].)

There are several points you will want the participants to arrive at:

1. Many of us did not decide on a teaching career.
 - Because we were good at and enjoyed math, we fell into teaching it.
 - Although we may have known that many careers may have been open to us because of our competency in math, most of us either did not explore them; many of us may not have even been too aware of these other career options. Not so our students--today, most of our best math students are looking at non-teaching alternatives. (There is and there is expected to be an even greater shortage of math teachers.)
2. Most of us did not decide to teach at the community college level.
 - about half of us saw the community college as an improvement over teaching at the secondary school level.
 - the other approximate half of us wanted to teach at the senior college or university level.
3. Relatively few college instructors (perhaps with the exception of ex-high school teachers) have had any formal training in pedagogy.
 - if at all, we were taught how to teach the traditional (in terms of age, motivation, and background) high school student.
 - most college instructors model their teaching practices on instructors that they have had.

- generally we teach the way we were taught and we expect our students to respond and learn the way we responded and learned.
4. Because they follow the accepted university model, community colleges do not "reward" good teaching although they subscribe to this as an important goal.
 - promotions and tenure still usually reflect the old "publish or perish" dictum.
 - work on committees and for professional associations is given great weight.
 - there is rarely little support given to staff development (the difficulty of arranging this program is a case in point) activities.
 - most department meeting time is allocated to administrative rather than to substantive or pedagogical concerns.
 5. Largely as a result of these factors, what we do in our classrooms and how, as well as how well, remains a very private matter.
 - while this has many benefits to us personally, it also means that we rarely get to share the insights of our colleagues or develop familiarity with one another's effective methods of instruction.
 - this leads to a tendency not to vary our way of teaching.
 - it also adversely affects our own interest in and excitement about teaching and doing mathematics.
- After years of teaching at a community college, most of us take increasingly greater interest in our students than in the subject matter.
 1. Our intrinsic rewards come from students who do well.
 2. There is little need to continue our study of math.
 - remedial courses, for most instructors, are not seen as challenging any longer.
 - calculus courses become a sought-after plum.
 3. We begin to develop biases and prejudices as well as preferences for those students who seem most like ourselves.
 - The typical community college student does not resemble us.
 1. Most importantly, they tend not to like math. The vast majority in fact have negative emotional reactions to math--for all practical purposes, they are math anxious.

2. They tend not to exhibit the motivational involvement in education that characterized us and our peers.
3. They are becoming less and less "traditional":
 - there is an increase in the number of women students, especially in "reentry ones".
 - there is an increase in their average age; many students today are adults.
 - there is greater ethnic and racial diversity.
- The techniques and strategies that we know best may not be effective with today's student.

There is a need, therefore, to reexamine our pedagogy.

- Each of us is a repository for effective teaching techniques.
 1. There is no one way to teach math.
 2. Some ways are more effective than others because they:
 - are relatively less complex.
 - respond to students' feelings.
 - take into account students' learning styles.
 - address students' beliefs and misconceptions.
 3. We can learn from one another.
 4. We can profit from theories of learning and from psychology but we don't need to be counselors to be good, effective math teachers.
 5. We will be unlikely to be effective with all students, but we can be more effective with more of them.
 - students, for example, with severe anxiety cannot be dealt with in the classroom.
 - similarly, it may not be possible to motivate all students to engage in study and learning habits that are crucial to math competency.
 6. Experimenting with different approaches is personally rejuvenating.
- Effective teaching begins on the first day of class (see Chapter 6 [4]).
 1. Instructors need to prepare for the first day.
 - the tone set at the first class carries through the semester.

- What you do has to be comfortable. Certain very dramatic things (like learning names in advance of the first day) may not be your style.
 - Part of what you do to prepare should be designed to help you regain feelings of empathy with learners. Remember that most students are not taking your course (remedial or advanced) voluntarily.
 - Try to recall and relive those times in your life when being in a math course caused you discomfort.
2. Use the entire first class period to get acquainted.
- this suggests that you at least are taking the course seriously.
 - it's a good idea to start a conversation with your students by telling them about yourself.
 - tell students about where and when you will be available to them after class. (Do not imply, however, that the course is hard.)
 - have students introduce themselves.
 - do not discuss grading or testing; save this for sometime later.
3. Your objective is to create an open, supportive environment.
- you should not be overly concerned with getting them to like you.
 - your major goal is to get them to concentrate and to attend to what you have to say.
 - however, an important, subsidiary goal is to have them begin to understand that both you and mathematics are human.
4. You might also attend to creating an atmosphere of trust and mutual cooperation.
- students tend to see math in an adversary way.
 - try things that indicate you are really on their side and that you have faith in them, even expect them to work to the best of their ability and desire.
5. It's O.K. to introduce some content on the first day. This suggests again that you are serious and hardworking and expect them to be. Choose content that guarantees that students have a success experience.

6. Try something new!

- like no other suggestion, this one helps you to get excited and your own excitement is a stimulus to others.
- a new tactic, something they may not have heard before engages students and is not threatening to them.

● From the start, encourage questions.

1. Ask students lots of questions; rhetorical ones won't do.

- this sets an example that questioning, even in math classes, is O.K.
- ask some easy questions to provide an opportunity for "quiet" students to answer.
- don't ask caustic or sarcastic questions (like, "why are you daydreaming?"). Again, ask only questions that have a real answer that will further the lesson or questions designed to clear up a problem (like, "what is there that you find confusing about this explanation?").
- ask questions that elicit a response. Questioning should not be used to maintain discipline.

2. Let students know they can and should ask you questions.

- you are therefore beholden to answer them all.
- not all student questions need to be answered in full during class. However, all questions need to be acknowledged and ones that imply a specific student difficulty dealt with out of class.

3. Never direct questions to a specific student.

- always ask for volunteers.
- similarly, never ask a particular student to do work at the blackboard.
- there are exceptions. You may ask a student who used a novel or unusual approach to demonstrate it if she/he is willing to, but keep in mind that most students get confused when more than one way of doing something is introduced.

4. Give them a "think about" question at the end of class.

● Always be prepared.

1. Too often we feel we don't have to prepare because we've taught it so often before.

- sometimes this leads us into neglecting to key the appropriate groundwork.

- it frequently leads us into coming up with (poor) examples--ones that are more complicated and messy than they need to be to demonstrate a particular point.
2. Many times the best way to personalize a topic is in its introductory phases.
 - examples that are "solved" by intuition are a particularly effective strategy; these take more time to set up than less intuitive problems.
 - the way a topic is presented can, in fact, turn off many students, especially those in remedial courses who "heard it all before". Novel ways of introducing a concept or operation will at least engage student attention.
- Be willing to back track.
 1. The syllabus is seductive.
 - we experience the pressure to get through our syllabus, sometimes no matter how many (or how few) of the students get there with us.
 2. Since math is a cumulative subject, however, it is really important that students understand what they are doing at every step.
 - this involves us in going back over "old" material.
 3. Time spent in review and in reteaching is time well-spent.
 - students cannot continue learning without the appropriate background. Ignoring this guarantees that you will just be going through the motions of teaching.
 - relearning anything is faster than learning it for the first time. It therefore doesn't take as long to review or reteach as it took to teach it initially.
 4. Sticking to the syllabus at any cost:
 - is boring; doesn't allow the instructor room for his or her creativity.
 - suggests that math is mechanical and inflexible.
 - curtails the opportunity for profitable interaction with students.

C. CALCULUS (30 minutes)

Purpose: the intent of including a program component on calculus is to demonstrate that anxiety reactions are not unique to remedial or introductory college-level math students and that the approach used for teaching fractions or signed numbers has applicability to more advanced mathematics. (See Chapter 8 [6])

- Turn the group to a discussion of calculus.

You might say, "Calculus has a mystique all its own. While in some ways it may be more fun to teach it than to teach fractions or algebra, learning it is not much different. Calculus involves conceptual leaps, new vocabulary--but vocabulary that has a confusing counterpart in everyday speech and a good working knowledge of basic algebraic and trigonometric manipulations."

1. Talk about how students approach calculus.
 - with fear and trepidation.
 - with the expectation that it is going to be difficult.
2. Few students take calculus voluntarily.
 - most of the students in the course are fulfilling a prerequisite.

- Discuss the difficulties of how the concept of limits is traditionally introduced.

1. It is usually introduced as preparation for teaching the concept of the derivative of a function.
2. In newer textbooks, the concept of derivatives is presented before the concept of limits.
 - derivatives are more graphical.
 - they create the need which the concept of limits fulfills.

- Set the context for the study of limits.

1. Spend time reviewing precalculus concepts.
 - this enables all students to begin the topic on an equal footing.
 - it also eases them into the course material.
2. After about two weeks of review, students should be ready to deal with the concept of a function.
 - since they've talked about the equation for a straight line, they now can focus on the slope of a curve.
 - the concept of the slope of the tangent is a distinct and interesting idea.
3. It is usually no problem to lead students through learning to deal with the slope of the secant lines.
 - at this point it is best to continue to avoid the possibility that "h" is negative; this postpones confusion and indeed may negate it entirely if the possibility of "h" being negative is best left until it is needed.

● Students are now ready for the limits concept.

1. Do not label it either by name or by implying that it is difficult.
 - labels at this point are unnecessary.
 - labels and the meaning of words detract students from focusing on the idea.
2. Instead of telling them you're going to introduce a new concept, raise some questions for students to consider:
 - for example, "what happens to the slope of the secant line when h gets smaller?", or
 - "geometrically, as h gets closer and closer to 0, what does the secant line look like?"
3. The question about whether or not h is ever 0 is the real point of difficulty.
 - encourage students to argue and to discuss this issue.
 - do not attempt to resolve the controversy.
4. Shift student attention to Zeno's paradox.
 - you might actually demonstrate by walking halfway across the room.
 - pose other questions for discussion ("If I were to get anywhere, where would it be? Is it ever possible with this sequence of steps to get beyond the wall? Is it possible there is a point before the wall where my walk ends?")
 - once there is agreement that you're always heading toward the wall they will have mastered the concept.
5. At this time, return to the question regarding the secant line and the tangent line.
 - Even though its heading toward the tangent and there's no place before the tangent where the secant stops, it seems it must make a last minute jump off the curve and land on the tangent line.
6. This jump is the "leap of faith" mastery of the concept requires.
 - review how they found the slope of the tangent line to the curve. (Do not yet discuss the possibility of h being negative.)
 - more examples will be needed before they accept the formal definition of the derivative.

7. This approach gives students:

- early exposure to the concept of derivatives.
- motivates them to learn more about limits.
- See Session #2 for things it is helpful to do with students when leaps of faith are required of them.

D. CLOSURE (5 minutes)

- Distribute copies of Chapter 8[6].
- Remind them they still have one formal session left, tell them the date, time, and place, and
- open up the suggestion of adding on some extra sessions to:
 - catch up.
 - go over some of the material in more detail.
 - demonstrate other ways to deal with students either generally or in terms of particular topics or strategies.

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MART: Mathematics Avoidance Reduction Techniques (III)

WORKSHOP SESSION #6

AGENDA

To prepare, review this section of the Leader's Guide

read Chapter 10 [8]

read Chapter 13 [11]

read Chapter 14 [12]

A. Welcome

B. Some Hints for Teaching Algebra

- what we do and why
- suggested ways to test effectively

C. Testing/Grading Practices

D. Selecting a Textbook and Getting Students to Read it

- evaluating textbooks
- what students need to know to use a text

E. Followup Activities: Where do we go from here?

- do we want more meetings?
- do we want to look into establishing a math clinic?
- do we have unanswered research questions?

F. End of Program Housekeeping

- Thank yous

Materials for Distribution to Workshop Participants

- Chapters 10 [8], 13 [11], 14 [12]

A. WELCOME (5 minutes)

As participants arrive,

- ask them to sign in.
- ask them to hold their comments to the end.
 1. There's a lot of new material to cover
 2. There will be opportunity for formal feedback

B. ALGEBRA (20 minutes)

Purpose: to highlight how math anxiety interferes with learning and to provide some helpful hints for presenting algebra to students.

- Without a mastery of algebra, further mathematics is not possible.
(This is not intended to be a complete discussion of how algebra can be taught. Rather, it pinpoints 2-3 points of difficulty for students.)
 1. Math anxious people are not comfortable with algebra.
 2. Students in college-level math courses frequently demonstrate an insufficient mastery of algebra.
- A major source of confusion comes from the use of letters to stand for numbers.
 1. In remedial courses or introductory algebra ones, stating that x stands for an unknown raises the question of how you can talk about something you don't know.
 2. A few minutes spent "demystifying" x can be very productive.
 - First discuss the idea of abbreviating the statement:
 $3 \text{ apples} + 5 \text{ apples} = 8 \text{ apples}$
by:
 $3a + 5a = 8a$
which looks more like algebra.
 - provide students with several additional examples of the same idea; e.g., $2p + 3p = 5p$, where p stands for pears.
 3. An important transition occurs when " p " stands for a number.
 - when we write $2t + 7t = 9t$, we first think of t standing for tree.
 - tell students to consider the possibility that t stands for 2.
 - have them practice reading 2 two's plus 7 two's equals 9 two's.

- check the numbers on the left and right of the equal sign to make sure they are correct.
- 4. Do similar exercises letting t stand for 3 and 12.
 - point out the statement would still make sense if t stood for 5 or for any other number.
 - tell them the " t " or any other letter is no longer suggestive and that by habit we have come to use a lot of x 's, y 's, and z 's.
- 5. Summarize the new concept: While letters in equations can be visualized as objects, these "objects" may also be numbers.
- 6. Tell them that in algebra very often they will be finding out what number(s) the letter(s) stands for.
- Follow this by using letters in equations.
 1. Generally, it is helpful to use suggestive letters like n for number and d for dimes.
 2. When equations are introduced start with easy ones that can be solved intuitively.
 - most students solve $n + 3 = 5$ intuitively.
 - provide them with a series of examples and then continue with examples like:
 - $n + 17 = 66$, $n + 7 = 22$ where they are likely to have to do some work.
 3. Help students realize that they are getting the answer by subtraction.
 - sequencing problems in this manner leads students to grasp the methodology at a level of intuitive understanding.
 - it also frequently arouses a desire on their part for more methodology.
 4. A similar sequence can be used for equations of the form of $ax = b$.
 - take the time to explain what the phrase "of the form" means.
- Help students visualize an equation as a balance scale.
 1. The overall goal is to get or to keep the scale in balance.
 2. Summarize the basic concept of the solution of equations as being based on the principle that "whatever you do to one side of the equation you must also do to the other side".
 3. Remind them that within this general guideline, they may use any approach they like.

- Tell students that all equations, the most simple to the most complex, are based on three things:

1. Having an objective--or knowing what you want to end up with so you can know what to get rid of;
2. Avoiding things you like the least (such as fractions), by getting them out of the equation; and
3. Keeping the scale in balance or doing the same thing to both sides of it.

C. TESTING/GRADING PRACTICES (45 minutes)

Purpose: to examine testing and grading policies to illustrate how we unintentionally exacerbate students' anxieties and to suggest modifications designed to reduce or alleviate the tension and threats tests impose. (See Chapter 13 [11].)

- Tests are the single most anxiety-provoking school activity.
 1. Math anxiety and test anxiety are independent phenomena.
 2. There are students who have both problems.
 3. Tests are threatening when they are viewed as representing a person's worth.
- There are many techniques for making tests a less threatening experience for students.
 1. Almost all the techniques, however, require the instructor to be flexible.
 2. It is possible to be flexible and, at the same time, to maintain high standards.
- Because tests are so much a part of academic life we often administer them without sufficient planning.
 1. Frequently there is a lack of congruence between why we say we test (our philosophy) and how we test (our practice).
 2. Tests are given for a variety of purposes:
 - to assign grades (i.e., to rank-rate students).
 - to predict performance in further math courses.
 - to predict performance (as prerequisites) in other curricula.
 - to diagnose individual's strengths and weaknesses.

- to assess how well we teach--to pace the material.
- to provide students with an additional learning experience.
- 3. It is important to understand our own priorities in order to modify our testing practices.
 - practices that don't further our aims can be rejected.
 - practices that are congruent with our goals can be considered.
- Tell students your testing grading policies.
 1. Do not do so at the first class session.
 2. Do it early in the term, providing them a picture of how often and when you will administer tests, what kinds of tests, make-up policies, and so on.
- Test dates should be announced well in advance.
 1. Surprise tests are the most threatening. They keep the class in a constant state of readiness, unable to concentrate on learning.
 2. Announced tests give students a chance to prepare.
- Give the first test early and on a small unit.
 1. This can be used as a sample test.
 2. It provides students with early feedback.
- Consider frequent quizzes.
 1. Despite their fear, students like a lot of tests.
 - this policy means each test is short and not worth too many points.
 2. Frequent tests keep students on their toes:
 - they encourage students to keep up.
 - they provide them with feedback.
 - they help them prepare for major exams.
 3. Frequent tests reduce anxiety because testing becomes habitual.
 4. Frequent tests don't necessarily take more of an instructor's time.
- Tell students what material they will be tested on.
 1. It helps them organize and prepare.
 2. It gives them more of a chance to succeed.
 3. It makes their fears more manageable.

- Return scored tests promptly.
 1. Fast feedback is a more powerful reinforcement.
 2. Students find that delayed feedback generates tension; they can deal with it if they expect it.
- Spend time discussing test-taking strategies.
 1. Students can benefit from knowing how to study and how to take tests.
 - do not cram; a little at a time is advantageous.
 - spend the most time on problems that are causing the most difficulty.
 - take advantage of extra help before the test.
 2. During the test:
 - students should allow themselves a limited time to panic.
 - students should find the problem easiest for them and start with it; it is not necessary to do the problems in order.
 - do not get hung up on any one problem; if you're having trouble, come back to it later.
 - students need to have trust in their intuition; their first answer is often the correct one.
- Personalize your scoring.
 1. Use the opportunity to note improvement or to request a meeting with students who do poorly.
 2. Identify patterns of error. This helps both you and the student understand what was wrong.
 - students can then be on the lookout for such errors.
 - instructors can get a feel for students' difficulties.
- Allow students sufficient time.
 1. A test is too long if most of the students can't finish it.
 2. A good guideline is that three-quarters of the class should be able to finish the test in the allotted time.
 3. Generally when students request more time it is not for an unreasonable amount.

- Timed vs. untimed tests.

1. Untimed tests are significantly less anxiety provoking.
2. Untimed tests permit students to demonstrate how much they know not how fast they can do it.
3. Most "real-life" situations are untimed or semi-timed.
4. Consider mixing timed and untimed tests.

- Don't give make-up tests.

1. A policy of make-ups gives a seal of approval to absences.
2. Do give all students a chance to improve their grades:
 - consider 3 of 4, 4 of 5 or some proportion of tests to be the basis of the final grade.
 - allow all students (including absent ones) an opportunity to take the test over + another test on the same material--or some variation.
3. Any strategy is worthwhile that suggests to students that mastery of the material is important.

- Consider giving partial credit.

1. All or nothing seems unfair to most students.
2. Most "real-life" situations are not all or nothing; people make mistakes without incurring severe penalties.
3. No credit for a careless mistake does not increase accuracy or neatness.
4. Partial credit permits instructors to differentiate between a careless mistake and more significant errors--such as those that indicate the student does not know the concept or principle involved.
5. Partial credit provides students encouragement to continue working to master that portion they don't yet have.

- There are special things to try with students who are severely test anxious.

1. Instructors may choose to work with some expert on campus.
2. Consider permitting the test anxious student to design the parameters of the situation under which he/she could best be tested. These might include:

- having unlimited time
 - taking a test in private
 - a take-home test.
- Students need to be told what to do with a returned test.
 1. Staring at it won't make it go-away.
 2. Disregarding it won't improve their knowledge.
 3. They need assistance in learning what they don't know.
 - Keep in mind that the purpose of making tests less threatening is that students are thereby freed to demonstrate what they know and what they don't know.

D. MATH TEXTS (15 minutes)

Purpose: to open up the question of how textbook writers make often unwarranted assumptions about the learner; to briefly explore criteria for textbooks; and to briefly suggest what students need to know about the book they are using. (Because of time, participants have the option of skipping the following section dealing with questions to ask...but some time devoted to what to discuss with students is highly recommended.)

- Suggest that the participants might want to form a group to propose criteria for the selection of textbooks...but for now let's
- Briefly list some questions to ask when evaluating a textbook.
 1. Is the format pleasing to the eye?
 - the relation between print and background is not only important aesthetically (as is the use of different size and type fonts), but can help or hinder the reader.
 - physical characteristics of the text help users maintain their place, locate an explanation or definition, and so on.
 - liberal use of headings and subheadings (as well as an adequate index and table of contents) is crucial for effective reading.
 2. Are there sufficient illustrations--pictures, diagrams, drawings?
 - these make the book seem less formidable, interesting, familiar, and human.
 - they provide a psychological break (a focused relaxation of concentration) when reading.
 - they greatly assist those students who learn best pictorially.

3. Is the writing style personal, clear, and sympathetic?
 - avoid texts that tell the reader in word or by deed (skipping over) that "It's obvious" or "clear" or "apparent".
 - jokes are not necessary, but an occasional light break is desirable.
 - interestingly, the older texts contained a wealth of mathematical anecdotes and historical references that seem to be almost completely missing from newer texts.
4. Does the text provide good and sufficient examples?
 - to the extent possible, better texts contain many illustrative examples that show the complete sequence of steps necessary for the solution and that summarize and review content, methods, and topics.
 - examples/problems should represent a wide range of difficulty.
5. Are the language and examples non-sexist and at a level students can understand?
 - sexist language (use of the he pronoun exclusively) is to be avoided.
 - sexist examples (content more likely to be appreciated by males--baseball batting averages, race car problems) reinforce sex-role stereotyping and give one sex a possibly unfair advantage.
 - clear, simple, and direct language is desirable. Overly sophisticated, complex, and ungrammatical language is not fair to students.
6. Is the text consistent with respect to notations and symbols?
 - the textbook writer should decide on a single set of symbols and use them consistently throughout the book.
 - similarly, the author should maintain an appropriate balance between mathematical clarity and mathematical rigor.
7. Does the text follow a developmental sequence that will make sense to students?
 - math's cumulative nature requires that texts follow a sequence from the less to the more difficult.
 - no steps should be omitted.
 - to a large extent, the book should be cross-referenced.

8. Is the text interesting?
 - texts that contain practical applications and historical perspectives help motivate students.
 - students appreciate the difficult definition, theorem, or proof if they see how it fits into the "big picture."
 9. What is the history of students response to the text?
 - any opinions from previous groups of students are useful in judging a text.
 10. How well does the text fit your personality, style, and sequence?
 - you and your students are the users. Texts that best match your opinions about what is important are also those that your classes will find easiest to learn from.
 - if you have to spend too much time supplementing the text or explaining it away, it is not the most appropriate one for you.
- As a transition, tell participants that there are things about math texts that students need to know. They tend not to be written in an easy-to-read manner.
 1. You might want to read them this example from the Algebra I book by Dressler:

"We know that the sum of two arithmetic fractions which have the same denominator is a fraction whose numerator is the sum of the given fractions. We use the same rule to add algebraic fractions which have the same non-zero denominator."
 2. This is a rather wordy but typical example of the writing found in math books.
 - the student who reads this at a normal reading speed will feel totally lost.
 - even for non-students it may require a moment of thought to realize the author is just talking about adding ordinary fractions.
 3. It would be an interesting exercise to have participants rewrite these two sentences in English, keeping them mathematically correct.
 - Tell your students that math texts are not meant to be read like novels. THEY MUST BE READ VERY, VERY SLOWLY.
 1. A speed of 1-3 pages an hour is quite good.
 2. Each word, phrase, and symbol must be understood before the student should proceed.

- Reading a math text is an active process.

1. Paper and pencil must always be on hand.
2. Example problems should always be worked out without looking at the solutions in the text. Even recopying the solution gives students practice and confidence.
3. Reference books (e.g., earlier texts) should be used in much the same way students use dictionaries. Students should find a basic book that each finds understandable.
4. Some students learn better if they read the text aloud; others when they rewrite it in an expository way; still others should be encouraged to draw pictures and diagrams.

- Math books tend to be cumulative.

1. Definitions and methods presented early on are not likely to be repeated. The author assumes once is enough.
2. It is thus necessary for the user to continually go back and refer to earlier sections of the book.
3. It is a good idea to have two books on the same subject.
 - math books are written, after all, by people who are naturally going to do better in some areas of explaining than they do in others.
 - sometimes the explanations in one book may be more helpful than in another.

E. FOLLOWUP ACTIVITIES (30 minutes)

Purpose: to discuss where your college goes from here. A very brief listing of some possibilities should be covered. (See Chapter 14 [12] which describes some math anxiety program for students.)

- Where do we go from here?

Briefly, there are several alternatives participants could consider:

1. Additional MART workshop-type meetings.
2. Establishing an out-of-class service for the severely math anxious student.
3. Answering unanswered empirical questions.

● Should we continue MART-like workshops?

The possibilities include:

1. Additional meetings for the original participants to:
 - cover in more depth some of the topics/issues already briefly introduced. (Which ones do we want to cover?)
 - discuss new topics of interest. (What are some examples?)
2. Another series for other faculty/staff. Possible new groups of participants include:
 - other mathematics faculty from this or other neighboring colleges.
 - faculty teaching math-based subjects (chemistry, physics, technologies) at this or other neighboring colleges.
 - counselors. (The program would have to be redesigned.)
 - math (lab) tutors or other personnel. (Again, the workshop content would have to be somewhat modified).

● Do we want to further investigate setting up a math clinic for students too anxious to deal with in the classroom? (See Chapter 14 [12])

Here, three of the options include (on a preliminary basis):

1. Contacting other established math clinics for information.
2. Forming a committee (of math faculty and Students Services' faculty) to research the question.
3. Inviting representatives from math clinics to come to the College to meet with us.

● Are there any unanswered empirical questions?

Some examples include:

1. Is math anxiety pervasive among our students? (Should we, for example, invest in purchasing copies of the MARS or MAS and administer them to students in: remedial courses? regular courses?)
2. Should we have a small demonstration (pilot) of a program involving students? (What do we want to demonstrate? With what kinds of students?)
3. Do we have any specific questions? For example, what are the effects of weekly tests vs. only a mid-term and final on math achievement in regular courses? in remedial courses?

Are there any sources of funding for special research and/or demonstration projects?

The answer is YES and NO.

1. Some grant money is still available but in generally lesser amounts.
2. There will be increasing competition for these more limited resources.
3. Your project has to be developed to reflect the priorities of the funding agency. For example, your College is eligible for (and most probably has been receiving) money from the State Education Department for projects that meet the guidelines of the Vocational Education Act (VEA). Projects supported with VEA funds generally have to focus on improving or demonstrating innovation services, programs, curricula, etc. in occupational education.
4. The grants officer at your College is the best source of information about funding possibilities. Request a meeting with him/her before too much time or effort goes into program (re)design.

F. CLOSURE

Since this is the last session of the MART program, there are some "housekeeping" details including:

- Distribute copies of chapters 10 [8], 13 [11], and 14 [12].
- Finally, but by no means of little importance:

THANK THEM FOR THEIR

- interest,
- participation,
- involvement, and
- willingness to attend.
- and THANK YOU for organizing and leading the program.
- We hope you all benefitted.

APPENDIX B
FACULTY FOLLOWUP SURVEY

Author's Note:

This questionnaire, which is a followup form of the Faculty Opinion Survey, is appended merely to complete the description of the 1981-82 MART (III) workshop program evaluation.

Unlike the Opinion Survey (see Appendix A, pages A10 through A14), the Followup Survey is not integral to the conduct of a workshop program. However, its use can provide the institution with some additional data about the program's impact on participating staff and faculty. Care should be exercised in the interpretation of pre-and post-program comparisons, however: as was demonstrated in the 1981-82 evaluation, the results will reflect the project's specific emphases. In other words, if the Project's focus differs (as did MART I from MART III), this instrument will not reflect those changes.

MART's effectiveness may be best assessed by the spontaneous and unsolicited reactions of participants and by the plans they generate for the future.

Center for Advanced Study in Education
Institute for Research and Development in Occupational Education
Graduate School and University Center
City University of New York

Project MART: FACULTY FOLLOW-UP SURVEY, 1981-1982

This opinionnaire, designed specifically for mathematics faculty, is similar to the one we asked you to complete at the beginning of the MART workshop series. It is intended to provide essential information which, along with other data, will enable us to evaluate the program's impact.

We guarantee anonymity and confidentiality of individual responses, but since we are interested in comparing your own responses before and after, we are asking you to identify yourself using the last four digits of your Social Security number. We appreciate your cooperation. Thank you.

Please enter the last four digits of your S.S. # here: _____

1. For how many years have you been teaching at the college level? _____

2. For each of the following statements, please indicate how you feel.	Strongly Disagree	Disagree Somewhat	Agree Somewhat	Strongly Agree
Generally there is only one good way to solve a math problem.				
Instructors should tell students the difficulties they themselves have in math.				
Math is a subject that one either knows or doesn't know.				
A good mathematical solution is elegant.				
Math is nice because it permits one to evaluate his/her own accomplishments.				
Math is neat and tidy.				
It is important to present single, clear, unambiguous solutions to math problems.				
Calculators should not be used in class.				
Students should keep working on a problem until they find the solution.				
Elementary school pupils should be discouraged from counting on their fingers.				
It doesn't matter how they do it as long as students get the right answer.				
Math is not creative.				
Some people have a "math mind" and some don't.				

3. Listed below are a number of statements. Please rate how important you feel each is in being an effective teacher of mathematics.

	Not Important At All	Somewhat Unimportant	Somewhat Important	Very Important
Knowledge of math history				
Knowledge of math folklore				
Arithmetic skill (calculations)				
Pedagogical technique				
Knowledge of the psychology of learning				
Preference for the specific subject				
Competence in the specific subject				
Knowledge of mathematical research issues				
Membership in professional organizations and associations				
Knowledge of students' feelings				
Knowledge of how math relates to other subject areas				

4. The following factors are believed to contribute to the solution of difficult math problems. Rate each.

	Not Important At All	Somewhat Unimportant	Somewhat Important	Very Important
Ability to do arithmetic calculations				
Knowing basic mathematical concepts				
Knowing how others did the problem				
Knowing what has failed in previous attempted solutions				
Logic				
Intuition or guessing				
Perserverance				
Practicing similar, but easier problems				

5. Many factors go into computing a student's grade. Please indicate how you feel about each.

	Not Important At All	Somewhat Unimportant	Somewhat Important	Very Important
Performance on the final				
Performance on the mid-term				
Performance on quizzes				
Participation in class				
Completion of homework				
Student's general ability level				
The effort the student makes				
The student's general attitude				
Your general impression				
Departmental policy				

6. Below are two math problems similar to ones that you might include on an exam in one of your courses. For each problem three typical solutions are given, all of which have incorrect answers. The maximum credit for each problem is 10 points. Please indicate how you would grade each solution by scoring each from 0 to 10.

Problem A: BILL HAS \$5.05 IN DIMES AND QUARTERS. IF THE NUMBER OF QUARTERS EXCEEDS TWICE THE NUMBER OF DIMES BY ONE, FIND THE NUMBER OF COINS OF EACH TYPE.

Solution #1

$$10 \text{ quarters} = \$2.50$$

$$20 \text{ quarters} = \$5.00$$

$$18 \text{ quarters} = \$4.50, 5 \text{ dimes} = 50¢, \text{ total} = \$5.00$$

$$15 \text{ quarters} = \$3.75, 13 \text{ dimes} = \$1.30, \text{ total} = \$5.05$$

ans. 15 quarters, 13 dimes

SCORE: _____ POINTS

Solution #2

$$\text{let no. dimes} = d$$

$$\text{let no. quarters} = 2d-1$$

$$\text{value of dimes} = 10d$$

$$\text{value of quarters} = 25(2d-1)$$

$$10d + 25(2d-1) = 505$$

$$10d + 50d - 25 = 505$$

$$60d = 480$$

$$d = 8$$

$$q = 15$$

ans. 15 quarters, 8 dimes

SCORE: _____ POINTS

Solution #3

$$\text{total coins} = \$5.05$$

$$\text{dimes} = 10N$$

$$\text{quarters} = 25(2N+1)$$

$$10N + 25(2N+1) = 505$$

$$10N + 50N + 25 = 505$$

$$60N = 540$$

$$N = 9$$

$$q = 19$$

ans. 19 quarters, 9 dimes

SCORE: _____ POINTS

Problem B: SOLVE FOR a : $-25 = \frac{7}{3}a - 11$

Solution #1

$$-25 = \frac{7}{3}a - 11$$

$$-11 \quad -11$$

$$-36 = \frac{7}{3}a$$

$$\frac{3}{7}(-36) = \frac{7}{3}a \left(\frac{3}{7}\right)$$

$$a = 5\frac{1}{7}$$

SCORE: _____ points

Solution #2

$$3 \cdot -25 = \frac{7}{3}a - 11 \cdot 3$$

$$-75 = 7a - 11$$

$$+11 \quad +11$$

$$-64 = \frac{7a}{7}$$

$$a = -9\frac{1}{7}$$

SCORE: _____ points

Solution #3

$$-25 = \frac{7}{3}a - 11$$

$$+11 \quad +11$$

$$3 \cdot -14 = 3 \cdot \frac{7}{3}a$$

$$\frac{42}{7} = \frac{7a}{7}$$

$$a = 6$$

SCORE: _____ points

7. In general, how much did MART help you understand, or stimulate you to consider:

	Not at all Helpful	Not very Helpful	Somewhat Helpful	Very Helpful
the needs of the math anxious student				
your grading criteria				
your testing strategy				
your teaching techniques				
your students' reactions to you				
your expectations for your students				
how to identify the math anxious student				
your understanding of how much your students know				
the way you teach fractions				
the way you teach algebra				
the way you teach word problems				
the way you teach signed numbers				
how much you as a mathematician take for granted				

8. Please indicate one MART-generated idea/technique that you would actually consider trying with your students:

9. How useful did you find talking with your colleagues?

☐ a waste of time ☐ somewhat useful
☐ not very useful ☐ very useful

10. Compared with other workshop-type programs in which you participated or heard about from others, how would you rate MART? Overall, MART was:

	Not at all	Not very	Somewhat	Very
Worthwhile				
Relevant				
Interesting				
Thought provoking				
Well planned				
Well organized				

11. What specific suggestions do you have for improving:
the workshop discussion format? _____

the organization and sequence of topics? _____

the specific content covered? _____

the training and preparation of the Discussion Leader? _____

12. Please use the rest of this page, or the reverse side for additional suggestions
and recommendations.

THANK YOU FOR COMPLETING THIS FORM

APPENDIX C
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